IN THE NAME OF GOD

Flight Mechanics II

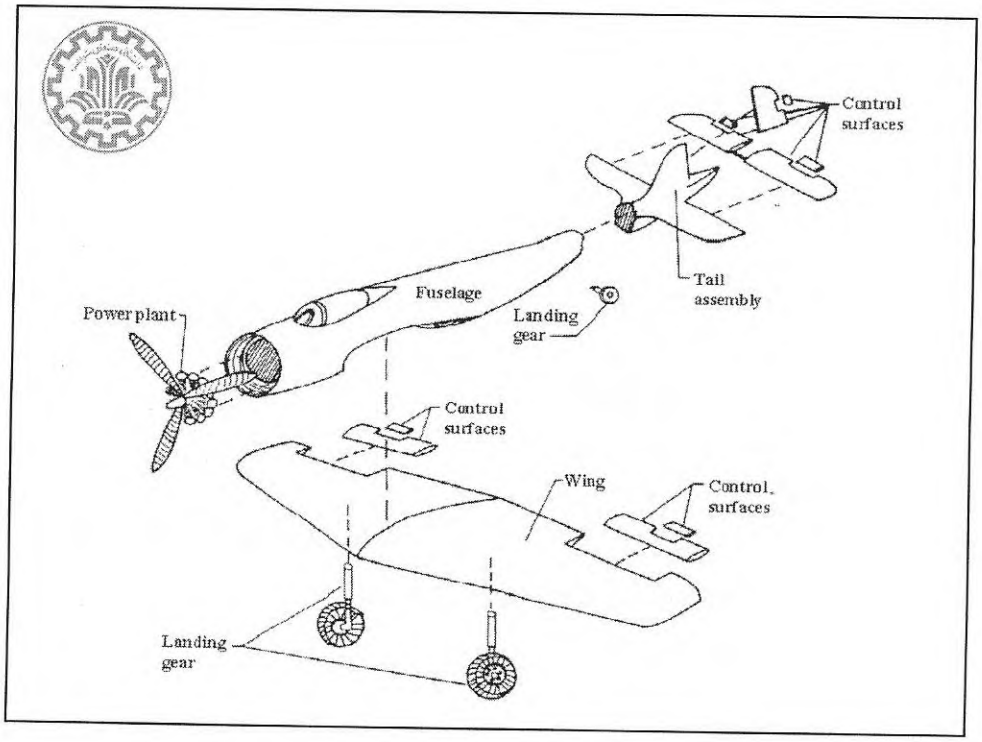
SYLLABUS

	Chapter in Text
I. Introduction and Review	**1&2**
II. Equation of Motion and Axis System	
Review of Aerodynamic Fundamentals	
II. Aerodynamic & Thrust Forces & Moments	**3**
Steady State Forces & Moments	
Perturbed State Forces & Moments	
Overview of General Signs for Aerodynamic Coefficients	
III. Stability & Control during Steady State Flight	**4**
Introduction	
Stability & Control Character. for Steady State, Straight Flight	
Stability & Control Character. for Steady State, Man. Flight	
Trim Comparisons for Conventional, Canard, and Three Surface Configurations	
Effects of the Flight Control System	
IV. Stability & Control during Perturbed State Flight	**5**
Longitudinal, Dynamic Stability & Response	
Lateral Directional, Dynamic Stability & Response	
Sensitivity Analysis	
V. Flying Qualities & Pilot Ratings	**6**
Flying Qualities & Pilot Ratings	
Military & Civilian Flying Quality Requirements	
Longitudinal Flying Quality Requirements	
Lateral Directional Flying Quality Requirements	

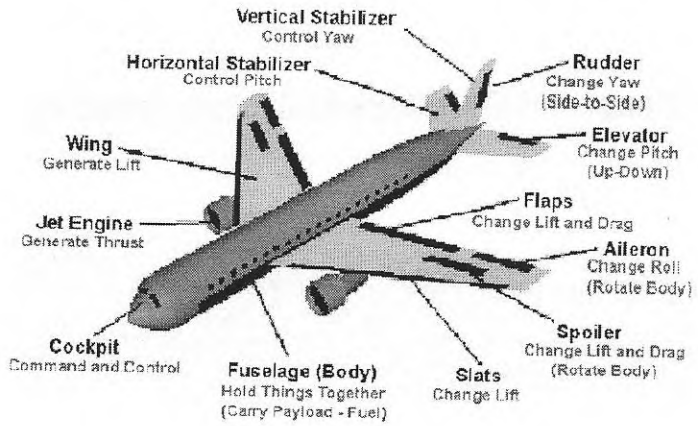


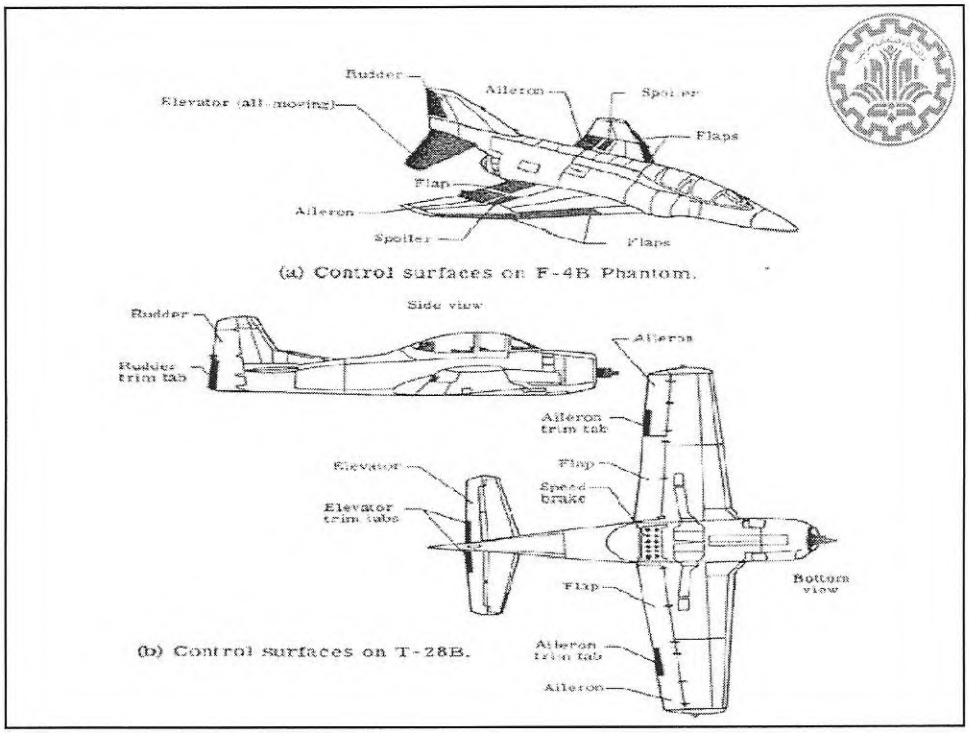
Additional References

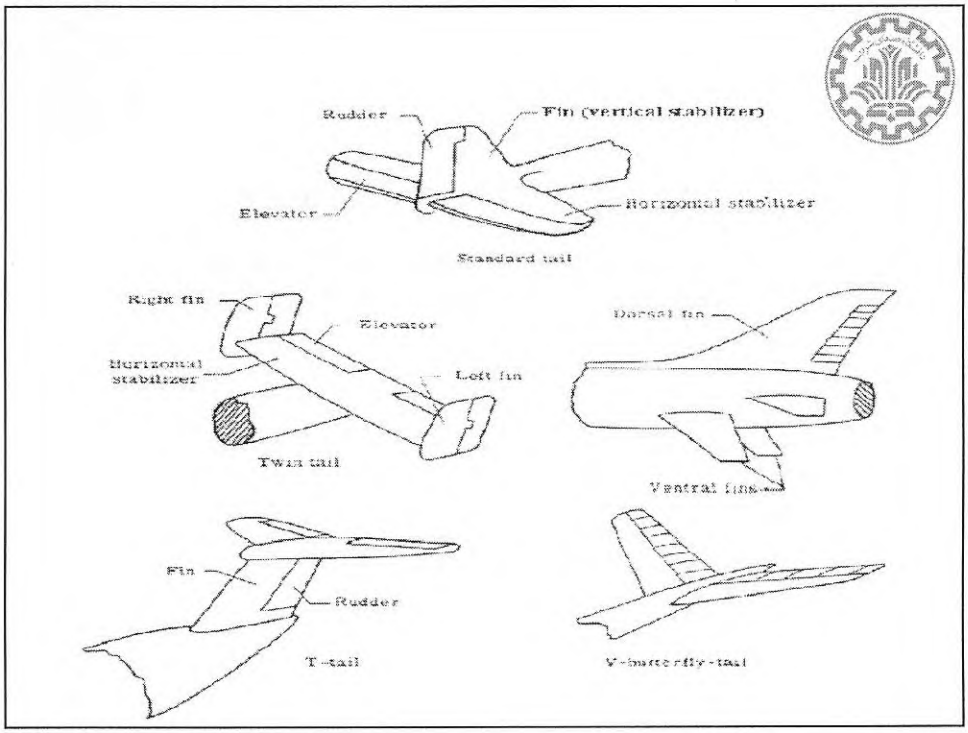
Hancock, G.J., *Introduction to the Flight Dynamics of Rigid Airplane*, 1995.
Etkin, B. and Reid, L.D., *Dynamics of Flight, Stability & Control*, 1996.
Schmidt, L.V., *Introduction to Aircraft Flight Dynamics*, 1998.
Nelson, R.C., *Flight Stability & Automatic Control*, 1989.
Phillips, W.F., *Mechanics of Flight*, 2004.



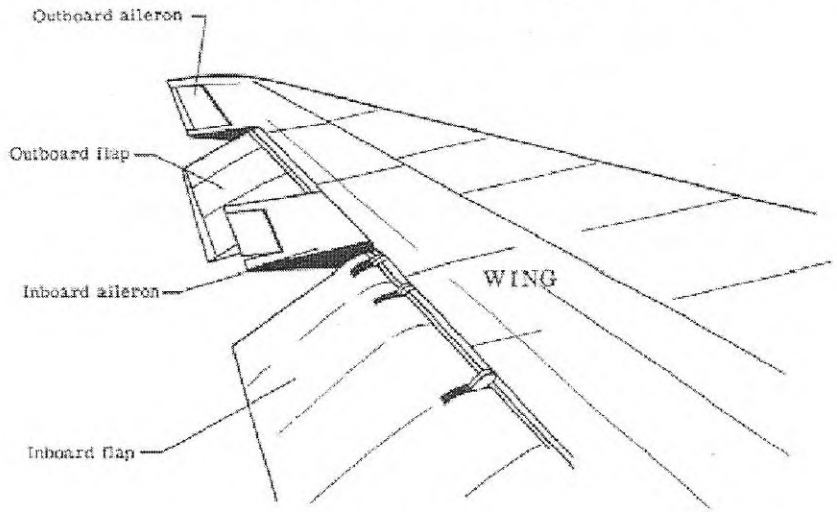
Anatomy of an Aircraft

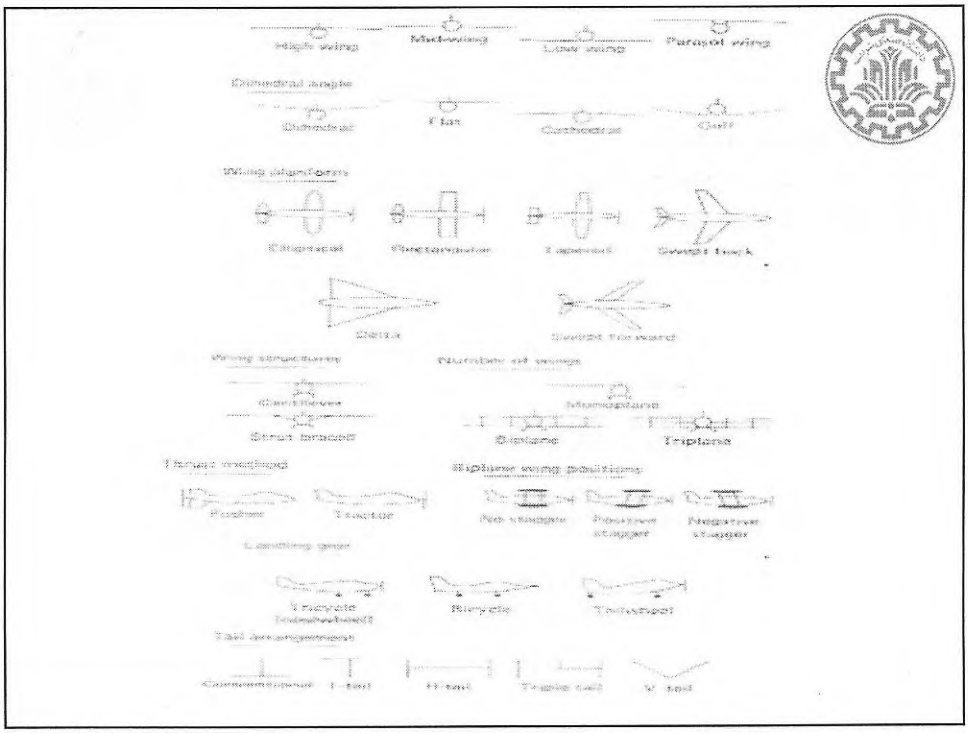


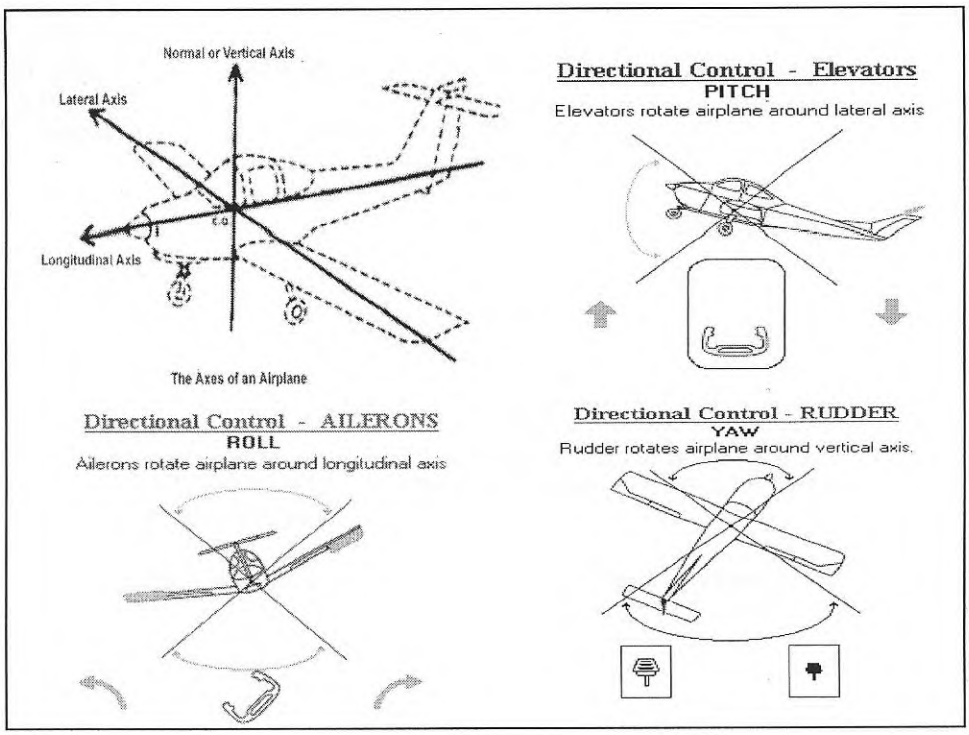

Rudder
Elevator (all-moving)
Aileron
Spoiler
Flaps
Flap
Aileron
Spoiler
Flaps
(a) Control surfaces on F-4B Phantom.
Side view
Rudder
Rudder trim tab
Aileron
Aileron trim tab
Elevator
Elevator trim tabs
Flap
Speed brake
Flap
Bottom view
Aileron trim tab
Aileron
(b) Control surfaces on T-28B.

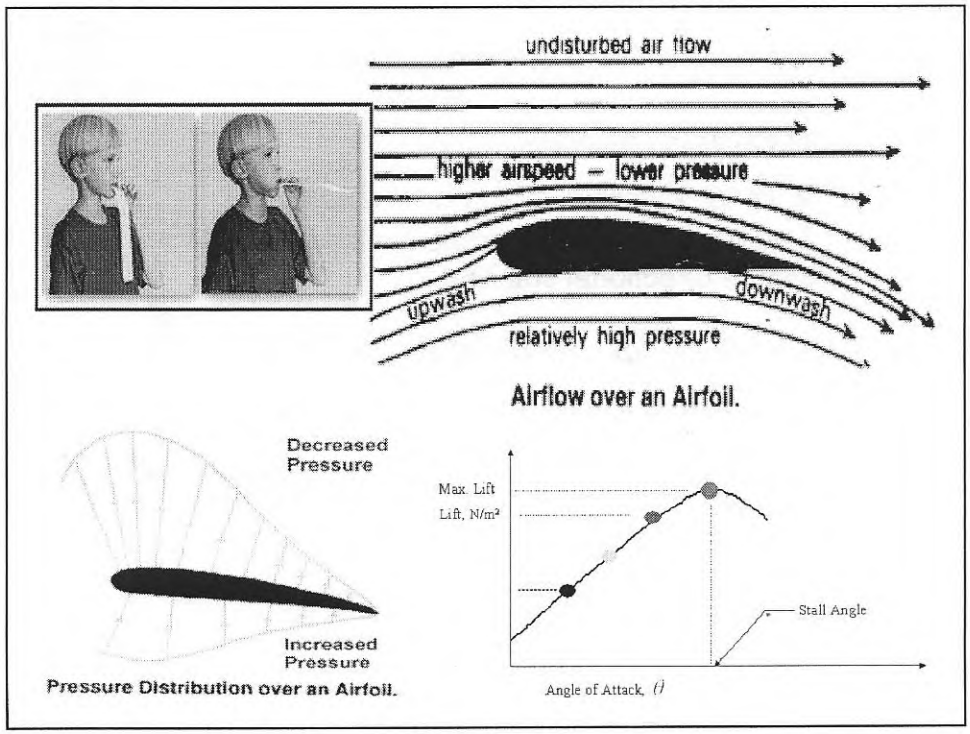

Rudder
Fin (vertical stabilizer)
Elevator
Horizontal stabilizer
Standard tail
Right fin
Elevator
Horizontal stabilizer
Left fin
Twin tail
Dorsal fin
Ventral fins
Fin
Rudder
T-tail
V-butterfly-tail

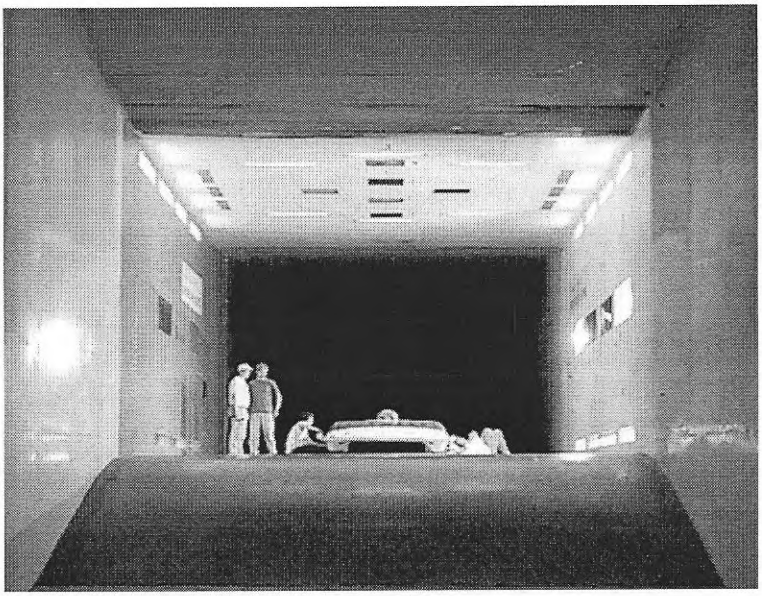
Jet Aileron Configuration

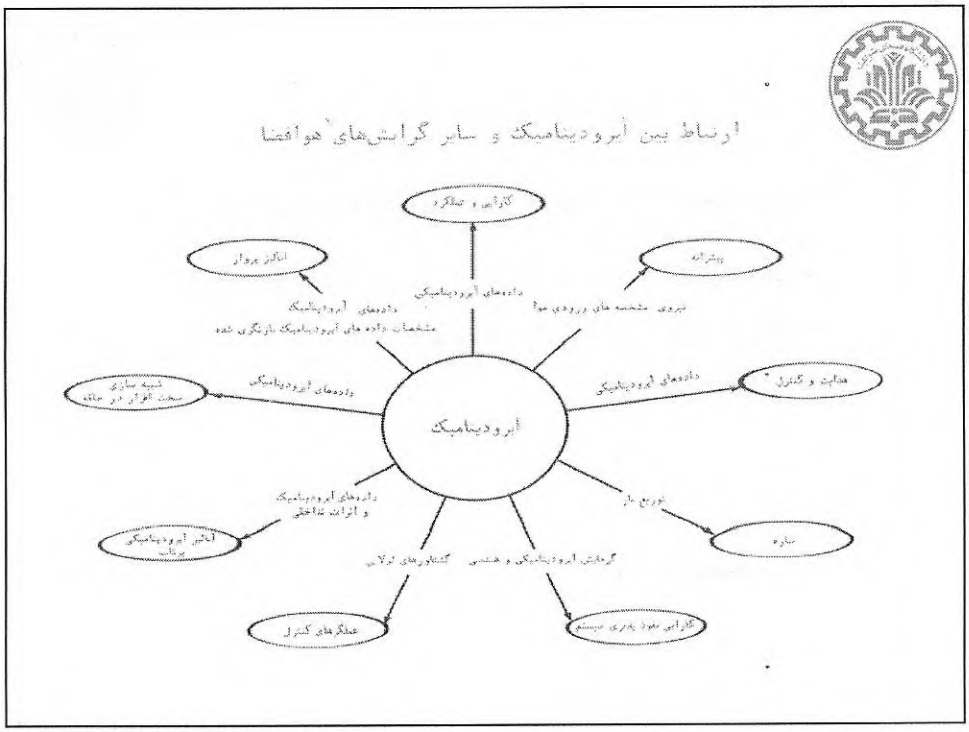





Normal or Vertical Axis
Lateral Axis
Longitudinal Axis
The Axes of an Airplane
Directional Control - Elevators
PITCH
Elevators rotate airplane around lateral axis
Directional Control - AILERONS
ROLL
Ailerons rotate airplane around longitudinal axis
Directional Control - RUDDER
YAW
Rudder rotates airplane around vertical axis.


undisturbed air flow
higher airspeed – lower pressure
upwash
downwash
relatively high pressure
Airflow over an Airfoil.
Decreased Pressure
Increased Pressure
Pressure Distribution over an Airfoil.
Max. Lift
Lift, N/m²
Stall Angle
Angle of Attack, (i)




ارتباط بین آیرودینامیک و سایر گرایش‌های هوافضا
آیرودینامیک
داده‌های آیرودینامیک
بیرونی مشخصه های ورودی مواد
پیشرانه
داده‌های آیرودینامیک
مشخصات داده های آیرودینامیک بازنگری شده
انالیز پرواز
داده‌های آیرودینامیک
هدایت و کنترل
داده‌های آیرودینامیک
داده‌های آیرودینامیک
و ابزار تداخلی
تونل باد
سازه
گرایش آیرودینامیکی و هندسی
عملگرهای کنترل
گرایش مهندسی سیستم

In the name of GOD

Flight Mechanics II

سروکار داشتن

Stability and Control (S&C) deals with how well an A/C flies & how easily it can be controlled.

S&C با اینکه هواپیما چطور خوب پرواز کند و چطور به آسانی می‌توان آن را کنترل کرد سروکار دارد.

Flight Dynamic

Aeroelasticity | S&C | Performance

-Performance Deals with : range, max. speed, cruise speed, take off & landing.

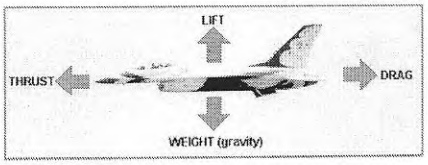
(1-I)

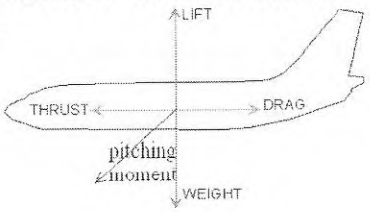
فرض می‌کنیم هواپیما یک جرم نقطه‌ای است.

- Considered A/C as a point mass
- Deal with C_L, C_D, W and T

- Performance deals with long term behavior of A/C motion only 3 degrees of freedom needed displacements.

Performance با ترم‌های بالانس نیروها حرکت A/C سروکار دارد و موقعیت را با دو درجه آزادی نشان می‌دهد





عکس العمل

-Stability & Control : Deals with the reaction of the A/C to internally or externally generated disturbance.

آشفتگی

- Internal disturbance : Changes in control surface,i.e. C.G changes, changes in A/C configuration (Flaps,landing gears, wing sweep angle).

S&C : با عکس العمل هواپیما نسبت به آشفتگیهای داخلی و خارجی سروکار دارد.

(1-II)

Internal disturbance caused by the pilot or Automatic control
⇨ Flying qualities

-External disturbances : turbulent, gust, altitude & temperature changes, etc. caused by the atmosphere.

An A/C must be designed so that it has a built in tendency to oppose or diminish all of the above motions.
⇨ ride quality & upset recovery quality

(2-I)



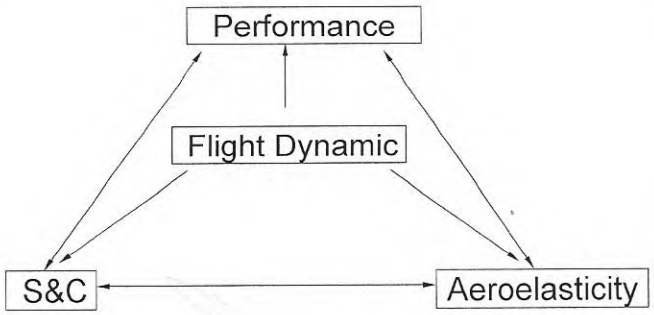
-Stability & Control deals with the short period behaviors of the A/C motion & need 3-linear displacement & 3 Angular displacement.

-Aero elasticity Deals with the integrity of A/C structure during both steady & unsteady Aerodynamic loads ;
Static & Dynamic Aero elasticity:

(2-II)

•Before 1950 all 3 Areas were considered to be independent of each other.

•After 1950.



(2-III)

- Stability & Control characters of A/C are referred to as the vehicle handling or flying & is a function of A/C mission. (like the A/C performance)

-Passenger A/C needs high level of stability.

-Fighter A/C needs high level of controllability for combat maneuvers.

A/C with little or no inherent Aerodynamic stability can be flown. They are unsafe, unless provided with stability Augmentation (SAS).

(3-I)

غیرنظامی

For civilians A/C , FAR (Federal Air regulation) part 23 gives regulation for light A/C ; W<= 12500 lbs

FAR part 25 gives regulation for transport A/C; W > 12500 lbs

For Military A/C MIL-F-8785.

(3-II)

معلوم کردن کیفیت

To quantify stability & control parameters need a common reference system.

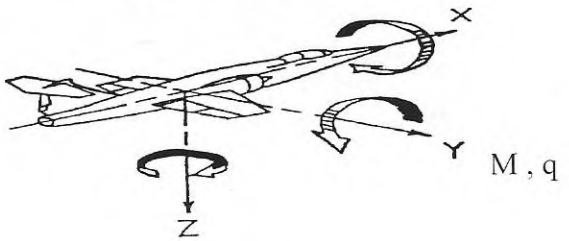
معلوم کردن کیفیت پارامترهای SBC به یک سیستم مرجع عمومی نیاز دارد.

Use body Axes system

X - longitudinal axes (plane of symmetry)

ℓ – Rolling Moment

P – Angular velocity (roll rate)

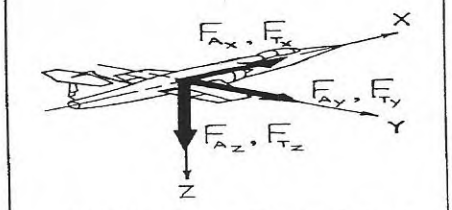


(3-III)

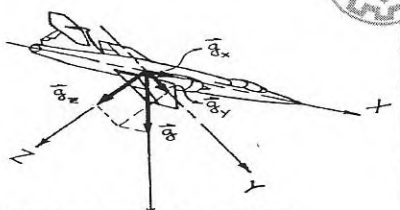
Y- Lateral Axis (positive to the right side of the A/C).
M- Pitching Moment.
q- Pith rate.

Z- vertical Axis ⊥ to the plane of symmetry.
Down is positive According to NACA's establishment.
n- Yawing Moment (positive to the right when viewed from the cockpit).
r- Yaw rate.

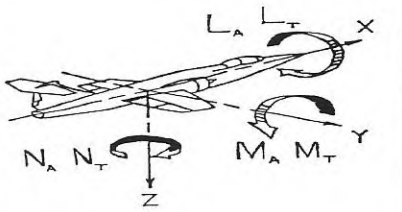
(4-I)



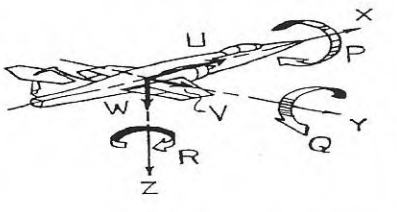
Aerodynamic and Thrust Forces



Acceleration of Gravity



Aerodynamic and Thrust Moments



Linear and Rotational (Angular) Velocities

Definition of Stability & Controllability

Two Kinds of Stability
- Static Stability
- Dynamic Stability

- Static stability: Tendency of an A/C to return to its equilibrium state after a disturbance.

پایداری استاتیکی: تمایل یک هواپیما به بازگشت به مرحلهٔ تعادل خودش بعد از یک آشفتگی را گویند.

(4-II)

Positive static stability.

Negative static stability: Tendency of an A/C to continue its movement when displaced from condition of equilibrium.

پایداری استاتیکی منفی: تمایل یک هواپیما به ادامه حرکت وقتی که از شرایط تعادل جابجا شود.

Neutral static stability: Neither positive nor negative.



Positive Static Stability | **Negative Static Stability** | **Neutral Static Stability**

(4-III)



- To have a stable equilibrium position, the vehicle must develop a restoring force and/or moment which tends to bring the vehicle back to the equilibrium condition.

برای داشتن یک موقعیت تعادل پایدار، وسیله باید یک نیرو و یا ممان بازگرداننده فراهم کند تا وسیله را به شرایط تعادل برگرداند.

. . Static Stability → Stick fixed stability
. . Static Stability → Stick free stability

(5-I)



•*Dynamic Stability:*

تعادل دینامیکی:

- Resultant motion of an A/C with time after being disturbed from trim.

نتیجه حرکت یک هواپیما با زمان پس از آنکه از حالت trim خارج شود.

A/C can be statically stable but dynamically unstable.

هواپیما می تواند استاتیکی پایدار باشد اما دینامیکی ناپایدار باشد

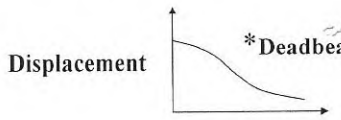
If an A/C is dynamically stable it must be statically stable too.

ولی اگر یک هواپیما دینامیکی پایدار باشد باید همچنین استاتیکی پایدار باشد.

(5-II)

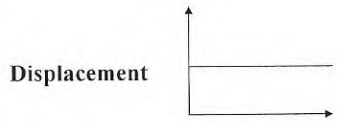
• Positive Dynamic stability: Amplitude of the resulting motion ↓ with time (Oscillatory: Non-Oscillatory)

A) Non Oscillatory Motion

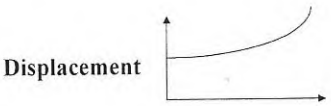


*Deadbeat (if the A/C subsides the disturbance without oscillation.)

Positive static & Dynamic



Neutral static & Dynamic



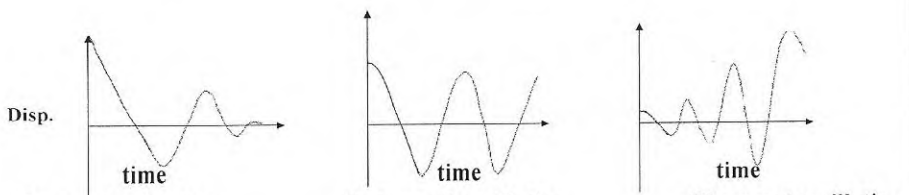
Negative static & Dynamic

(6-I)

B) Oscillatory Motion

Reduction of disturbance with time indicates that the energy is being dissipated →(positive damping).

If energy is being added to the system → (negative damping)



Damped oscillation (+static & Dynamic)

Undamped oscillation (+static & neutral Dynamic)

Divergent oscillation (+static & - Dynamic)

(6-II)

• If an A\C has neg. damping ⇨ dynamically unstable. Must use SAS

SAS – An electromechanical device which senses the undesirable motion & moves the appropriate control to damp out the motion
(small control movement)

-Degree of dynamic stability is very important to the pilot & designer.

Dynamic stability is usually specified by the time it takes a disturbance to be damped to half of the initial amp. or the time it takes for the initial amplitude. of the disturbance to double $T_{1/2}$ or T_{double} .

(7 – I)

اگر هواپیما دمپ منفی داشته باشد دینامیکی ناپایدار است و باید از SAS استفاده کند.

SAS - یک شیوهٔ الکترومکانیک که حرکت نامطلوب را حس می‌کند و کنترل مناسب را برای دمپ کردن آن حرکت انجام می‌دهد. (حرکت کنترلی کوچک)

درجه پایداری دینامیکی برای خلبان و طراح بسیار مهم است.

پایداری دینامیکی معمولاً به وسیلهٔ زمانی که یک آشفتگی به نصف دامنهٔ اولیه دمپ می‌شود یا زمانی که دامنهٔ اولیه آشفتگی دوبرابر می‌شود تعیین می‌شود ($T_{1/2}$ یا T_{double})

- If the pilot attempts to correct for a disturbance by moving a control surface, and the input is out of phase with the oscillatory motion, the action would increase the motion (pilot-induced oscillation, PIO).

Factor which contribute to PIO are: insufficient aerodynamics damping, insufficient control system damping, and pilot reaction time.

(7-II)

اگر یک خلبان برای اصلاح آشفتگی تلاش کند و یک سطح کنترل را حرکت دهد و یک نازخارجی با حرکت نوسان وارد کند و این عمل باعث افزایش حرکت شود، (خلبان نوسان وارد می‌کند.) PIO

عواملی که در PIO شرکت می‌کنند: دمپ کردن آیرودینامیکی نامناسب، دمپ کردن سیستم کنترلی نامناسب، و زمان عکس العمل خلبان.

- Trim: condition of equilibrium ⇨ A/C moments in pitch, roll and yaw are all=0

Trim is accomplished by pilot effort, trim tabs, etc.

-Hands off condition

(7 – III)

Controllabity:

The ability of the A/C to respond to control movements.

A/C with strong positive stability is very difficult to control since it has a strong residence to being disturbed from the trim condition.

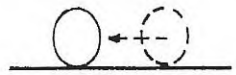
Same is true for strong negative stability too.



Requires considerable effort to maneuver

(8 – I)

Neutral Stability



Easy to move ⇨ peak in controllability

- A/C which need to be very controllable (fighter or aerobatic) have near neutral stability.

A/C with high controllability need to have near neutral stability, but for cross-country flying the pilot wants to have positive stability ;A trade-off exists between stability & controllability which depends on the airplane's mission.

(8-II)

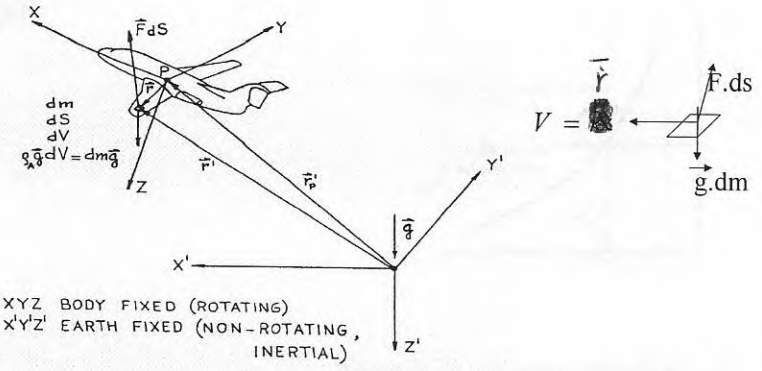
هواپیماهایی که به کنترل پذیری زیاد نیاز دارند (جنگنده و آکروباتیک) نزدیک پایداری خنثی هستند.

A/C با کنترل پذیری بالا نیاز دارد که نزدیک پایداری خنثی باشد اما در پروازهای طولانی خلبان می‌خواهد یک پایداری مثبت داشته باشد.
یک مبادله میان پایداری و کنترل پذیری است که به ماموریت هواپیما بستگی دارد.

IN THE NAME OF GOD

Flight Mechanics II

General Equation of Motion for a Rigid A/C



Assume Earth fixed (non – rotating) is an inertial system.

A/C is considered to be made up of dm mass particles.

r' is a position vector.

(9 – I)

فرض می‌کنیم کره‌ی زمین ثابت (غیر چرخشی) یک سیستم اینرسی است.

A/C فرض می‌شود که از ذرات جرم dm ساخته شده است.

r یک بردار مکان است.

⟹

A/C is rigid Mass particles remain at constant distance from each other except, rotor, turbine wheels, etc.

Newton's Second Law:

$\sum$ External forces acting on a body = Time rate of change of Momentum.

(9 – II)

For translational motion, for an element dm

$$\vec{df} = \vec{a}\,dm = \vec{\ddot{r}}\,dm = \vec{\ddot{r}}\,dm$$

$\vec{df}$: force on an element dm

- For rotational motion

Moment of df about θ' = time rate of change of angular momentum of element dm

OR

H: angular momentum $\quad \sum M = \frac{dH}{dt}$

$$F_x = \frac{d(mu)}{dt} \qquad F_y = \frac{d(mv)}{dt} \qquad F_z = \frac{d(mw)}{dt}$$

F_x, F_y, F_z: component of forces along x, y, z axes (aerodynamic, propulsion, gravity)

u, v, w : component of velocity along x, y , z

(10-I)

Moment Equation:

$$L = \frac{dH_x}{dt} \qquad M = \frac{dH_y}{dt} \qquad N = \frac{dH_z}{dt}$$

H_x, H_y, H_z: component of moment of momentum along x, y, z axes

(10 – II)

Roskam notation :

Forces aching on the elemental mass dm

body forces , gravitational forces ρgdv , & surface forces F

$$\int_s \vec{F} ds + \int_v \rho_A \cdot \vec{g}\, dv = \frac{d}{dt} \int_v \rho_A \cdot \frac{d\vec{r}}{dt} \cdot dv$$

Angular Momentum:

$$\frac{d}{dt} \int_v \vec{r} \times \rho_A \cdot \frac{d\vec{r}}{dt} \cdot dv = \int_v \vec{r} \times \rho_A \cdot \vec{g} \cdot dv + \int_s \vec{r} \times \vec{F} \cdot ds$$

Total mass of the A/C:

$$m = \int_v \rho_A \cdot dv$$

(11 – I)

Assumption:

- g is the only body force
- Mass distribution is const. with time : neglect fuel slush
- $\frac{dm}{dt} = 0$ (Not good for rocket) for about 30 – 60 sec .
- Origin of body fixed system at C.G.
- Rigid A/C
- Earth is assumed to be fixed
- XZ plane is the plane of symmetry

(11 - II)

فرضیات:
- g تنها نیروی بدنه است.
- توزیع جرم با زمان ثابت است.
حرکات داخل آن را در نظر نمی گیریم.
- $\frac{dm}{dt} = 0$ (برای راکت خوب نیست)
برای حدود 30 تا 60 ثانیه
- مبداء سیستم بدنه ثابت CG است.
- زمین ثابت فرض می شود
- صفحهٔ xz صفحهٔ تقارن است.
- هوا بی صدا است.

$\frac{dm}{dt} = 0$ for about 30 – 60 sec, typical of time duration over which A/C response are evaluated

Airplane or Rocket Type	Gross Weight (lbs)	Maximum Fuel Weight (lbs)	Cruise Fuel Consumption (lbs/hr)	Mass Change for 60 sec. (lbs)	Mass Change for 60 sec. in Percent of Gross Weight
SST	675,000	291,000	90,000	1,500	.22
Fighter	54,000	17,600	5,940	99	.18
Light Twin	6,800	1,020	200	3.3	.05
Saturn 5 First Stage	6,500,000	4,500,000	-----	1,800,000	27.6
Delta	112,000	100,000	-----	19,300	16.2

Mass dist is constant with time

i.e. A/C C .G. stays in the same place during a 60 sec interval.

- Fuel sloshing, shifting payload and wondering passengers are not considered.

(11 – A)

The velocity of diff. mass dm is

$$\vec{V} = V_p + \frac{d\vec{r}}{dt}$$

$$\vec{r} = \vec{r} + \vec{r}_p$$

V_P: Velocity of the center of mass of the A/C.

$\frac{dr}{dt}$: Velocity of element relative to the center of mass

$$\rightarrow \vec{r}_p = \vec{r} - \vec{r}$$

Multiply by:

$$m = \int_v \rho_A \cdot dv \Rightarrow \therefore \int_v \vec{r}_p \cdot \rho_A \cdot dv = \int_v \vec{r} \cdot \rho_A \cdot dv - \int_v \vec{r} \cdot \rho_A \cdot dv$$

(12 –I)

Since $\bar{r}$ is measured from the center of mass by definition of C.G

چون $\bar{r}$ از مرکز جرم توسط تعریف CG محاسبه می‌شود.

$$\Rightarrow \therefore \int_v \vec{r} \cdot \rho \cdot dv = 0 \qquad \Rightarrow \vec{r}_p = \frac{1}{m} \cdot \int_v \rho_A \cdot \vec{r} \cdot dv$$

$$\therefore \frac{d}{dt} \int_v \rho_A \cdot \frac{d\bar{r}}{dt} \cdot dv = \frac{d}{dt} \cdot \frac{d}{dt} \int_v \rho_A \cdot \bar{r} dv = \frac{d^2}{dt^2} \int_v \rho_A (\bar{r} + \bar{r}_p) dv$$

$$= \frac{d^2}{dt^2} \int_v \rho_A \cdot \bar{r}_p dv = \int_v \rho_A \cdot \frac{d^2 r_p}{dt^2} dv = m \frac{d\bar{V}_p}{dt}$$

(12 - II)

Where $\vec{V}_p = \frac{d\vec{r}'_p}{dt}$ velocity of the A/C center of mass

$$F = \sum \delta F = m\frac{d\vec{V}_P}{dt}$$

$$F_A = i\overrightarrow{F_{A_X}} + j\overrightarrow{F_{A_Y}} + k\overrightarrow{F_{A_Z}} \quad or$$

$$m\frac{d\vec{V}_p}{dt} = m\vec{g} + \vec{F}_A + \vec{F}_T$$

mg: gravity forces FA: Aerodynamic forces FT: Thrust force

For the Angular Momentum Equation →

$\int_v \vec{r}.\rho_A.dv = 0$ & $\vec{r}' = \vec{r}'_p + \vec{r}$ & Linear Momentum Equation

$$\frac{d}{dt}\int_v (\vec{r}'_p + \vec{r}) \times \rho_A \frac{d}{dt}(\vec{r}_p + \vec{r})dv = \int_v (r_p + \vec{r}) \times \rho_A g dv + \int_s (\overrightarrow{r_p} + \vec{r}) \times F ds$$

$$\Rightarrow \frac{d}{dt}\int_v \vec{r} \times \frac{d\vec{r}}{dt}\rho_A dv = \int_s \vec{r} \times F ds \qquad (13 - I)$$

$\int_s \vec{r} \times F.dS = \overrightarrow{M}_A + \overrightarrow{M}_T$; $\vec{M}_A = i\vec{M}_{Ax} + j\vec{M}_{Ay} + k\vec{M}_{Az}$

M_A: Total Aerodynamic Moment Vector

M_T: Total Thrust Moment Vector

$$\Rightarrow \quad \frac{d}{dt}\int_v \bar{r} \times \frac{d\bar{r}}{dt}\rho_A dv = \overline{M}_A + \overline{M}_T$$

Volume integral is time dep. , eliminate it

حذف کردن

Another Method ; For an elemental Mass dm

$$\delta M = \frac{d}{dt}\delta H = \frac{d}{dt}(r \times V)\delta m$$

$$V = V_p + \frac{d\vec{r}}{dt} = V_p + \omega \times \vec{r}$$

r: position of dm from c.g ω: angular velocity of the A/C

(13 - II)

For total A/C:

$$H = \sum \delta H = \sum \left(\vec{r} \times V_p\right) dm + \sum \vec{r} \times \left(\vec{\omega} \times \vec{r}\right) dm = \sum \vec{r} dm \times V_p + \sum \vec{r} \times \left(\vec{\omega} \times \vec{r}\right) dm$$

r is measured from the center of mass $\rightarrow \sum r\, dm = 0$

$$H = \sum \left[\vec{r} \times \left(\vec{\omega} \times \vec{r}\right)\right] dm$$

$$\vec{\omega} = pi + qj + rk$$

$$\vec{r} = xi + yj + zk$$

p: roll rate; q: pitch rate; r: yaw rate

$A \times (B \times C) = (A.C)B - (A.B)C$

$\mathbf{r = x\,i + y\,j + z\,k}$

$$\rightarrow \mathbf{H = (p\,i + q\,j + r\,k) \sum (x^2 + y^2 + z^2)\, dm - \sum (x\,i + y\,j + z\,k)(px + qy + rz)\, dm}$$

(14 - I)

The Scalar Components of H are

$$H_x = p\sum (y^2 + z^2) dm - q\sum xy dm - r\sum xz dm$$
$$H_y = -p\sum xy dm + q\sum (x^2 + y^2) dm - r\sum yz dm$$
$$H_z = -p\sum xz dm - q\sum yz dm + r\sum (x^2 + y^2) dm$$

$$I_{xx} = \int_v (y^2 + z^2) dm \qquad I_{xy} = \int_v xy dm \qquad I_{yy} = \int_v (x^2 + z^2) dm$$

$$I_{xz} = \int_v xz dm \qquad I_{zz} = \int_v (x^2 + y^2) dm \qquad I_{yz} = \int_v yz dm$$

(14 - II)

$$H_x = pI_{xx} - qI_{xy} - rI_{xz}$$
$$H_y = -pI_{xy} + qI_{yy} - rI_{yz}$$
$$H_z = -pI_{xz} - qI_{yz} + rI_{zz}$$

- To eliminate time dep., switch coordinate system. Write in term of xyz instead of xẏz
- XYZ is non-inertial rotating, cannot apply Newton's law. It can be shown that using ;

$$\left.\frac{d\vec{A}}{dt}\right|_I = \left.\frac{\delta\vec{A}}{\delta t}\right|_B + \vec{\omega}\times\vec{A}$$

- I: Inertial Frame (xẏz)
- B: Body Fixed Frame (xyz)
- ω: Angular velocity of system xyz relative to xẏz
- (Angular velocity of A/C)

(15 – I)

Now:

Or
$$F = m\frac{d\vec{V_P}}{dt} = m\left(\frac{\delta\vec{V_P}}{\delta t} + \vec{\omega}\times\vec{V_P}\right) = m\left(\dot{\vec{V}}_p + \vec{\omega}\times\vec{V_P}\right)$$

$$m\left(\dot{\vec{V}}_p + \vec{\omega}\times\vec{V}_p\right) = m\vec{g} + \vec{F}_A + \vec{F}_T$$

Linear momentum in body fixed system

Now Angular Momentum:

$$\frac{d}{dt}\int_v \vec{r}\times\frac{d\vec{r}}{dt}\cdot\rho_A dv = \int_v \frac{d\vec{r}}{dt}\times\frac{d\vec{r}}{dt}dm + \int_v \vec{r}\times\frac{d^2\vec{r}}{dt^2}dm$$

$$\Rightarrow \int_v \vec{r}\times\frac{d}{dt}(\dot{\vec{r}} + \vec{\omega}\times\vec{r})dm = \int_v \vec{r}\times\left(\frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\frac{d\vec{r}}{dt}\right)dm$$

$$= \int_v \vec{r}\times[\ddot{\vec{r}} + \dot{\vec{\omega}}\times\vec{r} + 2\vec{\omega}\times\dot{\vec{r}} + \vec{\omega}\times(\vec{\omega}\times\vec{r})]dm$$

(15 - II)

Assume Rigid A/C → $\vec{r} = const$ & $\dot{\vec{r}} = \ddot{\vec{r}} = 0$

$$\int_v \vec{r} \times [\dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})] dm = \vec{M}_A + \vec{M}_T$$

Angular momentum in body fixed coordinates

$\dot{\omega}$: Angular acceleration of A/C

- Equations of motion for a rigid A/C in the body fixed axes are:

Linear momentum

$$m(\dot{\vec{V}}_p + \vec{\omega} \times \vec{V}_p) = m\vec{g} + \vec{F}_A + \vec{F}_T$$

Angular momentum

$$\int_v \vec{r} \times [\dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})] dm = \vec{M}_A + \vec{M}_T$$

(16-I)

$$\vec{M}_T = il_T + jM_T + kN_T \qquad \vec{M}_A = il_A + jM_A + kN_A$$

$$\vec{V}_p = iu + jv + kw \qquad \vec{g} = ig_x + jg_y + kg_z$$

$$\vec{\omega} = \vec{i}p + \vec{j}q + \vec{k}r$$ p: roll , q : pitch , r : yaw

l_A: rolling, M_A: pitching, N_A: yawing

$$\vec{\omega} \times \vec{V}_p = \begin{pmatrix} i & j & k \\ p & q & r \\ u & v & w \end{pmatrix} = i(qw - vr) - j(pw - ur) + k(pv - uq)$$

(16-II)

The Component of Linear Moment Equations are:

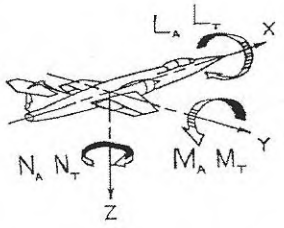
Body fixed system

X – Comp $$m(u - vr + wq) = m\vec{g}_x + \vec{F}_{Ax} + \vec{F}_{Tx}$$

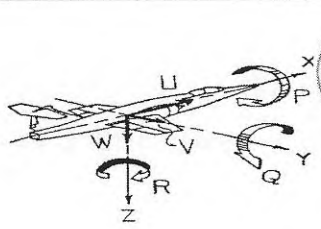
Y – Comp $$m(v + ur - wp) = m\vec{g}_y + \vec{F}_{Ay} + \vec{F}_{Ty}$$

Z – Comp $$m(w - uq + vp) = m\vec{g}_z + \vec{F}_{Az} + \vec{F}_{Tz}$$

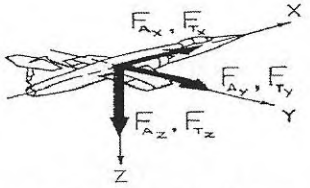
(17 - I)



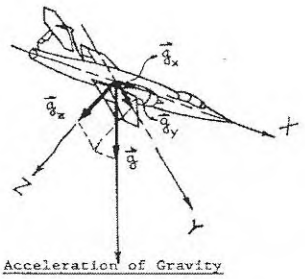
Aerodynamic and Thrust Moments



Linear and Rotational (Angular) Velocities



Aerodynamic and Thrust Forces



Acceleration of Gravity

(8-IV)

Now the angular momentum:

$$A\times(B\times C)=(A.C)B-(A.B)C$$

$$\int_v \vec{r}\times[\dot{\vec{\omega}}\times\vec{r}+\vec{\omega}\times(\vec{\omega}\times\vec{r})]dm$$

$$=\int_v\{(\vec{r}.\vec{r})\dot{\omega}-(\vec{r}.\dot{\vec{\omega}})\vec{r}+\vec{r}\times[(\vec{\omega}.\vec{r})\vec{\omega}-(\vec{\omega}.\vec{\omega})\vec{r}]\}dm$$

$$=(i\dot{P}+j\dot{Q}+k\dot{R})\int_v(x^2+y^2+z^2)\,dm-\int_v[(\vec{i}x+\vec{j}y+\vec{k}w)\times(x\dot{P}+y\dot{Q}+z\dot{R})\,dm]$$

$$+\int_v\vec{r}\times(\vec{\omega}.\vec{r})\vec{\omega}\,dm$$

$$=i(\dot{P}I_{xx}-\dot{Q}I_{xy}-\dot{R}I_{xz})+j(\dot{Q}I_{yy}-\dot{P}I_{xy}-\dot{R}I_{yz})+k(\dot{R}I_{zz}-\dot{P}I_{xz}-\dot{Q}I_{yz})+$$

$$\int_v\vec{r}\times(\vec{\omega}.\vec{r})\vec{\omega}\,dm$$

I_{xy} , I_{xy} , … have great influence on the dynamic Stability , control & handling character. of A/C. For preliminary design have to use available data.

(17 - II)

$$\int_v \vec{r}\times(\vec{\omega}.\vec{r})\vec{\omega}dm=\int_v (ix+jy+kz)\times(ip+jq+kr)(px+qy+rz)dm$$

For a symmetric A/C aloud xz plane, Ixy = Iyz = 0

(Missiles can have xy plane as a plane as symmetry)

Ixz=0

Angular moment equation to body fixed axes:

X – Comp $\quad I_{xx}\dot{P}-I_{xz}\dot{R}-I_{xz}PQ+(I_{zz}-I_{yy})RQ=l_A+l_T$

Y – Comp $\quad I_{yy}\dot{Q}+(I_{xx}-I_{zz})PR+I_{xy}(P^2-R^2)=M_A+M_T$

Z – Comp $\quad I_{zz}\dot{R}-I_{xz}\dot{P}+(I_{yy}-I_{xx})PQ+I_{xz}QR=N_A+N_T$

To obtain to time history of motion U(t) → R(t) must solve the above six equation ; to get u , v , w ,p , q & r

Note that to solve the above equations we must have L , D , Y , M , l , N as f (t) & g_x , g_y , g_z too.

(18)

Spinning Rotors :

- Assumed A/C is a single rigid body
- Some portions of the A/C mass are spinning relative to the body axes; rotors of jet engine, propellers, helicopter blades.
- They exert gyroscopic moment on the body; can neglect for most cases, but not for helicopters.

h- Resultant relative angular momentum of all rotors.

$$\vec{h_b} = \sum \vec{h_i}$$

(19 - I)

W_{Ri} - rotational velocity.

$$\vec{h} = \sum_i I_{Ri}\,\omega_{Ri}$$

$$\vec{h} = ih_x + jh_y + kh_z$$

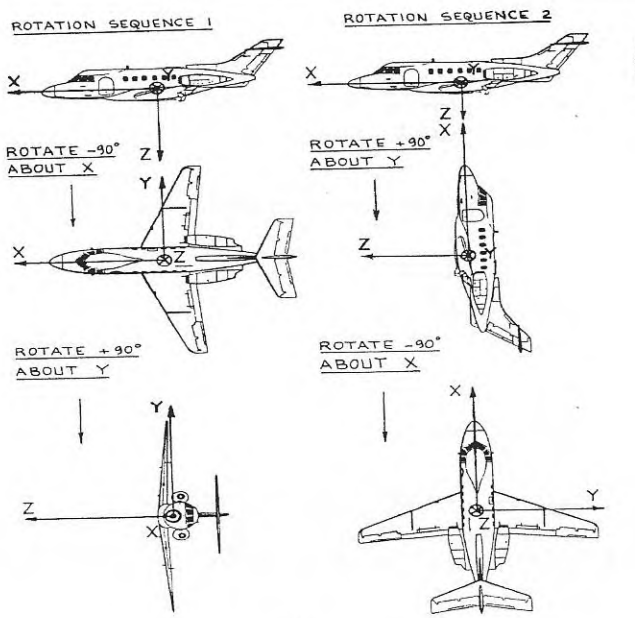
I_{Ri} - moments of each rotor relative to it's own axes.

For angular momentum:

$$\frac{d}{dt}\int_V \vec{r}\times\frac{d\vec{r}}{dt}\rho.dv + \frac{d}{dt}\vec{h} = \vec{M}_A + \vec{M}_B$$

$$\frac{d}{dt}\vec{h} = \vec{i}(qh_z - rh_y) + \vec{j}(rh_x - ph_z) + \vec{k}(ph_y - qh_x)$$

(19-II)



(20)

Position & Orientation : The Euler Angle

Using Newton's 2nd law in terms of a coordinate system fixed to A/C & Then made the inertia indep. of time which is good for complex geometry.

- Body coordinate system can not be easily used for describing the position & orientation of A/C. Used for onboard instrument report flight parameters.

⇨ It is more convenient to describe the a/c position and orientation in terms of Earth Fixed Coordinate.

- The position of the A/C relative to Earth is normally described in terms of latitude, longitude, and elevation.

(20-A-I)

IN THE NAME OF GOD

Flight Mechanics II

Position & Orientation : The Euler Angles

Using Newton's 2nd law in terms of a coordinate system fixed to A/C & Then made the inertia indep. of time which is good for complex geometry.

- Body coordinate system can not be easily used for describing the position & orientation of A/C. Used for onboard instrument report flight parameters.
- ⇨ It is more convenient to describe the a/c position and orientation in terms of Earth Fixed Coordinate.
- The position of the A/C relative to Earth is normally described in terms of latitude, longitude, and elevation.

(20-A-I)

مکان وجهت: زوایای اویلر

قانون دوم نیوتن از یک سیستم مختصات ثابت برای A/C استفاده می کند و سپس یک انرژی غیروابسته به زمان کمکی هندسه های پیچیده بسیار خوب است می سازد

سیستم مختصات بدنه به آسانی نمی تواند برای شرح مکان وجهت A/C استفاده شود. از یک وسیلهٔ خارجی برای گزارش پارامترهای پرواز استفاده می شود

این بسیار راحت است که مکان وجهت A/C را در مختصات ثابت زمینی شرح دهند.

مکان A/C نسبت به زمین معمولاً به وسیلهٔ توصیفی عرض وطول وارتفاع شرح می دهند.

Over short periods, the distance traveled is small compared to R_{earth} ⇨ Can describe position in terms of a Cartesian System.

برای دورهٔ کوتاه، فاصلهٔ طی شده در مقایسه با شعاع زمین کوچک است

می‌توان مکان را با سیستم کارتزین شرح داد

For Cartesian system:

X-axes pointes **north**
Y-axes pointes **east**
Z-axes pointes **down**

⇨Position of the A/C is specified by the location of the origin of the body-fixed coordinate frame relative to the Earth fixed coordinate frame.

مکان A/C بوسیلهٔ مکان مبداء مختصات بدنهٔ ثابت نسبت به مختصات زمین ثابت مشخص می‌شود.

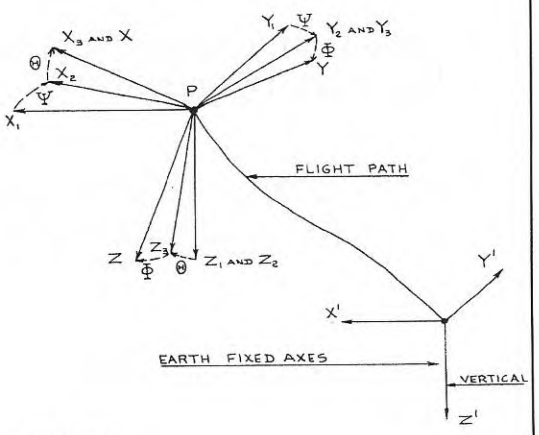
(20-A- II)

جهت یابی یک A/C نسبت به زمین را می‌توان با ترم های زوایای اویلر شرح داد.

⇨ The orientation (جهت یابی) of an A/C relative to Earth can be described in terms of Euler Angle.

The orientation of body-fixed coordinate frames relative to the Earth Fixed Coordinate frame is described in terms of 3 consecutive rotations (پیاپی، پی در پی) thru Euler angles.

جهت مختصات بدنهٔ ثابت نسبت به مختصات زمین ثابت در 3 ترم متوالی چرخش شرح داده می‌شود زوایای اویلر.



(20-B- I)

If X'Y'Z' translates parallel to itself until its origin coincides with P $\Rightarrow X_1Y_1Z_1$

" Ordered rotations are required to align the X'Y'Z' with the body fixed. "

Relative orientation of XYZ to X'Y'Z' can be described by 3 Consecutive rotations (Need 3 rotations to align X'Y'Z' with XYZ system)

ψ about Z_1 (heading or yaw angle) $\Rightarrow$ $X_2Y_2Z_2$ Rotate about Z_1

θ about Y_2 (attitude or pitch angle) $\Rightarrow$ $X_3Y_3Z_3$ Rotate about Y_2

Φ about X_3 (bank or roll angle) $\Rightarrow$ XYZ Rotate about X_3

(20-B-II)

Note Euler angles Φ, θ, ψ bank, elevation & azimuth or heading angle.

If the rotations are as;

Let: X_f, Y_f, Z_f : Earth fixed

X_b, Y_b, Z_b : Body fixed

1. Rotate X_f, Y_f, Z_f coordinate system about Z_f axis thru an angle ψ to $X_1Y_1Z_1$
2. Rotate $X_1Y_1Z_1$ about Y_1-axis thru θ to $X_2Y_2Z_2$
3. Rotate $X_2Y_2Z_2$ about X_2-axes thru Φ to X_b, Y_b, Z_b

The Euler angles are not yaw, pitch or roll angles.

- Because azimuth angle is rotated about Z_f : Yaw is rotated Z_b axes.
- Similarly the elevation angle is about Y_f axes, while pitch angle is about Y_b axes.

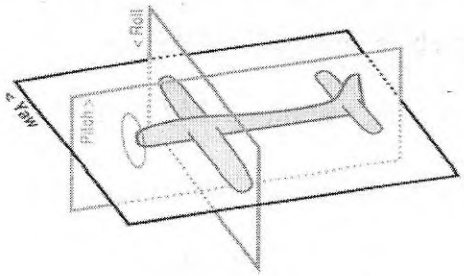
(20 – B – III)

Further roll, pitch, & yaw are orthogonal while Euler angles are not.

Can relate Euler to roll, pith & yaw but it is not one-to-one relationship.

Ex ; Can change bank angle without any roll input.

Ψ, θ, and Φ are referred to as the Euler Angles.



(20 – B –IV)

Flight Path Equation:

Now can determine the flight velocity components relative to the fixed frame.

Velocities in body axes $\vec{V}_p$ are u, v, and w

Velocities in X'Y'Z' are $\dot{x}=u_1 \quad ,\dot{y}=v_1 \quad ,\dot{z}=w_1$

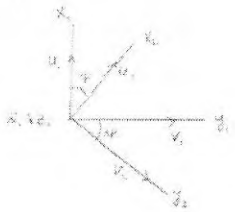
Now want to relate velocities in each rotation

1) Rotate about Z1 axes $\Rightarrow \psi$

$$u_1=u_2\cos\psi-v_2\sin\psi$$

$$v_1=v_2\cos\psi+u_2\sin\psi$$

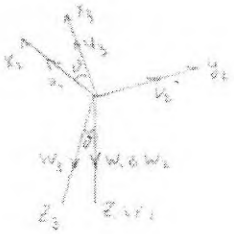
$$w_1=w_2$$



(22-I)

$$\rightarrow \begin{bmatrix} U_1 \\ V_1 \\ W_1 \end{bmatrix} = \begin{bmatrix} Cos\psi & -Sin\psi & 0 \\ Sin\psi & Cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_2 \\ V_2 \\ W_2 \end{bmatrix}$$

2) Rotate θ about y



$$u_2 = u_3 \cos\theta + w_3 \sin\theta$$

$$v_2 = v_3$$

$$w_2 = -u_3 \sin\theta + w_3 \cos\theta$$

(22-I)

$$\rightarrow \begin{bmatrix} U_2 \\ V_2 \\ W_2 \end{bmatrix} = \begin{bmatrix} Cos\theta & 0 & Sin\theta \\ 0 & 1 & 0 \\ -Sin\theta & 0 & Cos\theta \end{bmatrix} \begin{bmatrix} U_3 \\ V_3 \\ W_3 \end{bmatrix}$$

3- Rotate ϕ about X_3

$$\begin{bmatrix} U_3 \\ V_3 \\ W_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & Cos\phi & -Sin\phi \\ 0 & Sin\phi & Cos\phi \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

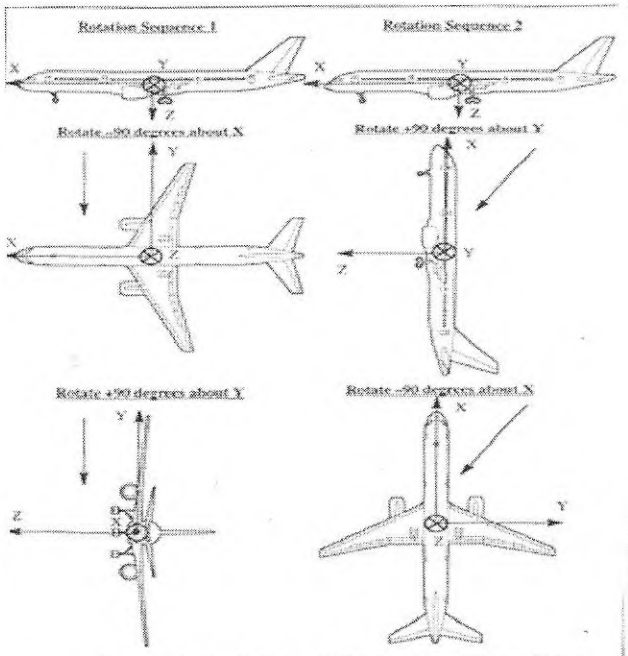
(22-I)

$$\begin{bmatrix} U_1 \\ V_1 \\ W_1 \end{bmatrix} = \begin{bmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{bmatrix} =$$

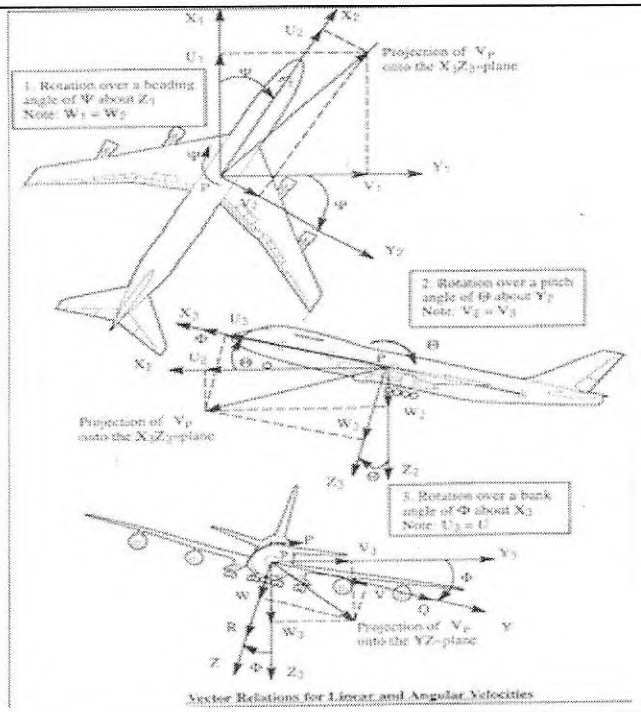
$$\begin{bmatrix} Cos\theta Cos\psi & Sin\phi Sin\theta Cos\psi - Cos\phi Sin\psi & Cos\phi Sin\theta Cos\psi + Sin\phi Sin\psi \\ Cos\theta Sin\psi & Sin\phi Sin\theta Sin\psi + Cos\phi Sin\psi & Cos\phi Sin\theta Sin\psi - Sin\phi Cos\psi \\ -Sin\theta & Sin\phi\ Cos\theta & Cos\phi\ Cos\theta \end{bmatrix}$$

$$\times \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

(22-II)



Demonstration that Finite Angular Rotations do not Behave as Vectors



Relationship between Velocity Components in X'Y'Z' & XYZ

- Now X'(t) , Y'(t) , Z'(t) are found by Integration
- Note that ϕ , ψ and θ are f (t) too. & are related to ω

$$\overline{\omega} = p\,\hat{i} + q\,\hat{j} + r\hat{k} = \vec{\dot{\psi}} + \vec{\dot{\theta}} + \vec{\dot{\phi}}$$

Angular rate about y_2

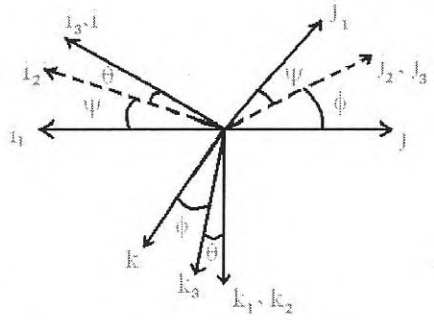
$$\vec{\dot{\psi}} = \hat{k}_1\dot{\psi} = \hat{k}_2\dot{\psi} \quad , \quad \vec{\dot{\theta}} = \hat{j}_2\dot{\theta} = \hat{j}_3\dot{\theta} \quad , \quad \vec{\dot{\phi}} = \hat{i}_3\dot{\phi} = i\dot{\phi}$$

Angular rate about z_1 ; Angular- rate about x_3

$$\Rightarrow \overline{\omega} = \hat{k}_2\dot{\psi} + \hat{j}_3\dot{\theta} + \hat{i}\,\dot{\phi}$$

(22-III)

Now from a similar transformation



Can show that:

$$p=\dot{\phi} - \dot{\psi}\ Sin\theta$$
$$q=\dot{\theta}\ Cos\phi + \dot{\psi}\ Cos\theta\ Sin\phi$$
$$r=\dot{\psi}\ Cos\theta\ Cos\phi - \dot{\theta}\ Sin\phi$$

(23-I)

Or:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -Sin\theta \\ 0 & Cos\phi & Cos\theta\ Sin\phi \\ 0 & -Sin\phi & Cos\theta\ Cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Body axis Roll Rate , $\overline{p}$, is not the same as $\dot{\overline{\phi}}$

Body axis Pitch Rate , $\overline{q}$, is not the same as $\dot{\overline{\theta}}$

Body axis Yaw Rate , $\overline{r}$,is not the same as $\dot{\overline{\psi}}$

(23-II)

Or:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & Sin\phi \tan\theta & Cos\phi \tan\theta \\ 0 & Cos\phi & -Sin\phi \\ 0 & Sin\phi\ Sec\theta & Cos\phi\ Sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\Rightarrow \theta = \int_0^t (q\ Cos\ \phi - r\ Sin\ \phi)\, dt$$

$$\psi = \int_0^t (q\ Sin\phi + r\ Cos\phi)\ Sec\theta\ dt$$

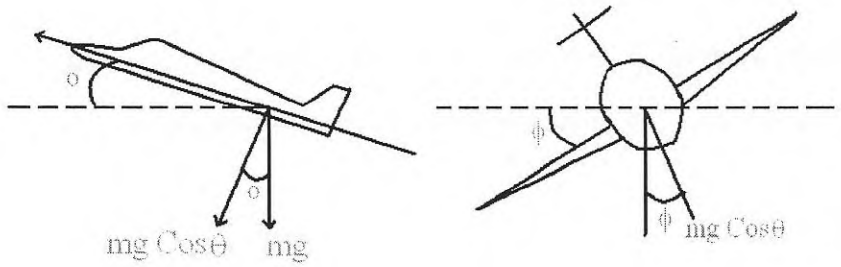
$$\phi = \int_0^t (p + q\ Sin\phi \tan\theta + r\ Cos\phi \tan\theta)\, dt$$

(23-III)

Gravitational & Thrust Forces

- Gravitational forces act thru the a/c C.G.
  Since body axis is fixed to the C.G. =>
  No moments.

نیروی گرانش بر روی CG هواپیما
از آنجا که بنابراین محور بدنه
نسبت به CG ثابت می ماند
← بدون ممان



(24-I)

$$\left(F_x\right)_{gravity} = -mg\ Sin\theta$$

$$\left(F_y\right)_{gravity} = mg\ Cos\theta\ Sin\phi$$

$$\left(F_z\right)_{gravity} = mg\ Cos\theta\ Cos\phi$$

$$g = \hat{i}g_x + \hat{j}g_y + \hat{k}g_z\ ;$$

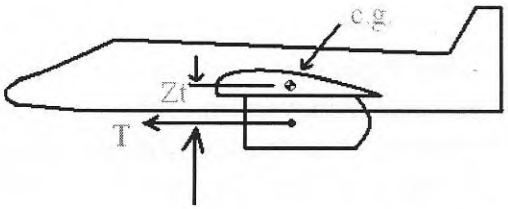
$$g_x = -g\ Sin\theta \qquad g_y = -g\ Cos\theta\ Sin\phi \qquad g_z = -g\ Cos\theta\ Cos\phi$$

Note : ψ Does not appear in the g Equations.
(Flat earth assumption)

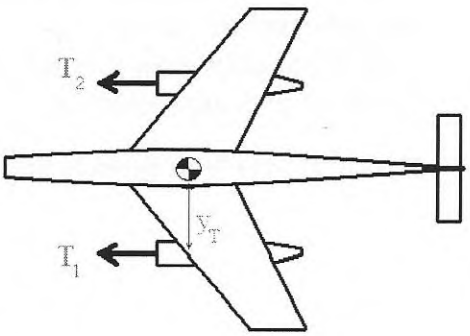
نکته : ψ نمی تواند در معادلات g ظاهر شود. (زمین صاف فرض شده است)

(24-II)

$$M_T = Z_T\ T$$



$$N_T = (T_1 - T_2)\ y_T$$



∴ Thrust forces creates Moment

(24-III)

Summery:

Two groups of Equations → Force & Moments

Two groups of Equations → Kinematic Equations

3 Linear Momentum: u , v ,w

$$\begin{cases} m(u - vr + wq) = -mgSin\ \theta + F_{A_x} + F_{T_x} \\ m(v + ur - wp) = mgSin\ \Phi Cos\ \theta + F_{A_y} + F_{T_y} \\ m(w - uq + vp) = mgCos\ \phi Cos\ \theta + F_{A_z} + F_{T_z} \end{cases}$$

3 Angular Momentum: p , q and r

$$\begin{cases} I_{xx}\ \dot{p} - I_{xz}\ \dot{r} - I_{xz}\ pq + (I_{zz} - I_{yy})\ rq = L_A + L_T \\ I_{yy}\ \dot{q} + (I_{xx} - I_{zz})\ pr + I_{xz}\ (p^2 - r^2) = M_A + M_T \\ I_{zz}\ \dot{r} - I_{xz}\ \dot{p} + (I_{yy} - I_{xx})\ pq + I_{xz}\ qr = N_A + N_T \end{cases}$$

body fixed axis (XYZ)

The above 6 Equations are called General Aircraft Equation of Motion

(25-I)

3 Kinematic Equations:

$$\begin{cases} \dot{\phi} = p + q\ Sin\phi\ \tan\theta + r\ Cos\phi\ \tan\theta \\ \dot{\theta} = q\ Cos\phi - r\ Sin\phi \\ \dot{\psi} = (\ q\ Sin\phi + r\ Cos\phi\ )\ Sec\theta \end{cases}$$

$$\Rightarrow \quad \dot{\phi}, \dot{\theta}, \dot{\psi} = f(p, q, r)$$

3 Flight Path Equations:

$$\begin{cases} \dot{x}' = U(Cos\theta Cos\psi + Sin\phi Sin\theta Cos\psi - Cos\phi Sin\psi \\ \qquad + Cos\phi Sin\theta Cos\psi + Sin\phi Sin\psi) \\ \dot{y}' = V(Cos\theta Sin\psi + Sin\phi Sin\theta Sin\psi + Cos\phi Sin\psi \\ \qquad + Cos\phi Sin\theta Sin\psi - Sin\phi Cos\psi) \\ \dot{z}' = W(-Sin\theta + Sin\phi\ Cos\ \theta + Cos\phi\ Cos\theta) \end{cases}$$

(25-II)

Equations are:

- Non-Linear
- Coupled
- 1st Order u, v, w
- 2nd Order θ, ϕ, ψ

(25-III)

Full Equation can be solved by → Numerical Methods
→ Analog Integrations

Solutions are useful for:
- -Simulators
- -Accident Investigations
- -Analyze Maneuvers
- -Spin Entry
- -Take-Off & Landing
- -A/C Response to Large Disturbances
- -Stalled Flight

A/C Motions for Practical Purposes are for Steady-State and/or Small Perturbation one.

Two special forms →
- S.S. Flight Condition
- Perturbed State Flight Condition

(26)

Steady State Flight:

Def n: A flight condition when all motion variables remain constant w.r.t time relative to a body axes system (XYZ).

with respect to

- ✓Steady rectilinear flight
- ✓Steady level turn
- ✓Steady symmetrical pull-up

$$\dot{\vec{V}}_p = 0 \;\&\; \dot{\vec{\omega}} = 0$$

(21 – I)

شرایط پروازی که در آن کلیهٔ پارامترهای پرواز در دستگاه مختصات بدنه نسبت به زمان ثابت باقی بماند

Strickly Speaking for S.S. Flight, one must fly at a constant altitude.

Since $\rho = \rho$ (h)

∴ Only steady level turn is S.S. Flight

Note: $\bar{q} = \frac{1}{2}\rho \; V_P^2$

∴ As ρ changes, $\bar{q}$ changes =>L, D, …. Varies

But as long as ρ does not vary more than 5% during 30-60 sec it is OK

$\Rightarrow \vec{V}_p = C_1 \qquad \vec{\omega} = C_2$ $\vec{V}_p$ Airplane Velocity, true airspeed

(27-I)

برای پرواز S.S باید در یک ارتفاع ثابت باشد

اما اگر مقدار ρ خیلی بیشتر از 5٪ در طول 30-60 تغییر نکند این درست است

- Linear Momentum: From Now on subscript 1 infers to S.S

$$X: m(\dot{U}_1 - V_1 r_1 + q_1 W_1) = -mg\sin\theta_1 + \vec{F}_{Ax1} + \vec{F}_{Tx1}$$

$$Y: m(\dot{V}_1 + U_1 r_1 - W_1 p_1) = mg\sin\Phi_1\cos\theta_1 + \vec{F}_{Ay1} + \vec{F}_{Ty1}$$

$$Z: m(\dot{W}_1 - U_1 q_1 + V_1 p_1) = mg\cos\Phi_1\cos\theta_1 + \vec{F}_{Az1} + \vec{F}_{Tz1}$$

- For Angular Momentum:

$$I_{xx}\dot{P}_1 - I_{xz}\dot{r}_1 - I_{xz}p_1 q_1 + (I_{zz} - I_{yy})r_1 q_1 = l_{A1} + l_{T1}$$

$$I_{yy}\dot{q}_1 + (I_{xx} - I_{zz})p_1 r_1 + I_{xz}(p_1^2 - r_1^2) = M_{A1} + M_{T1}$$

$$I_{zz}\dot{r}_1 - I_{xz}\dot{p}_1 + (I_{yy} - I_{xx})p_1 q_1 + I_{xx}q_1 r_1 = N_{A1} + N_{T1}$$

(21 – II)

Kinematics equations :

$$p_1 = \dot{\phi}_1 - \dot{\Psi}_1\sin\theta_1$$

$$q_1 = \dot{\theta}_1\cos\phi_1 + \dot{\Psi}_1\cos\theta_1\sin\phi_1$$

$$r_1 = \dot{\Psi}_1\cos\theta_1\cos\phi_1 - \dot{\theta}_1\sin\phi_1$$

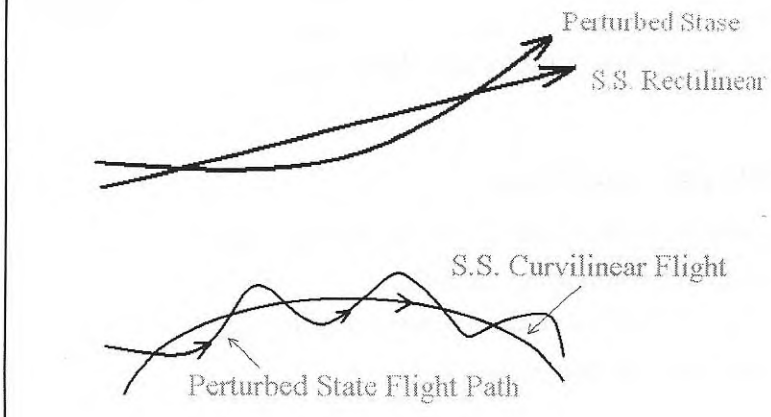
Perturbed State Flight:

Definition: Perturbed State Flight Condition is defined as one for which all motion variables are defined relative to a known S.S. Flight Condition.

∴ Total State of each motion variables = $\sum$ [S.S. (ref. or equilibrium flight) + Perturbed State quantity]

$u = u_1 + u$

(27-II)



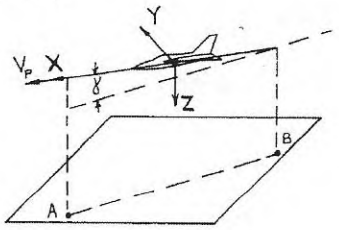
(27-III)

1)Steady state rectilinear flight

A flight where $\vec{\omega} = 0$, $p = q = r = 0$

$\therefore$ linear momentum

$$\left.\begin{aligned} -mg\sin\theta_1 + F_{A_{x_1}} + F_{T_{x_1}} &= 0 \\ mg\sin\phi_1\cos\theta_1 + F_{A_{y_1}} + F_{T_{y_1}} &= 0 \\ mg\cos\phi_1\cos\theta_1 + F_{A_{z_1}} + F_{T_{z_1}} &= 0 \end{aligned}\right\} \sum \vec{F} = 0$$



(28-I)

Angular momentum

$$\left.\begin{aligned} l_{A_1} + l_{T_1} &= 0 \\ M_{A_1} + M_{T_1} &= 0 \\ N_{A_1} + N_{T_1} &= 0 \end{aligned}\right\} \sum M = 0$$

No kinematics eq^{n}.

Used to study: cruise, shallow climbs, dives & glides

engine out flight, S.S flight with certain failed system

Will discuss later ; chapter 4

(28-II)

2)S.S turning flight

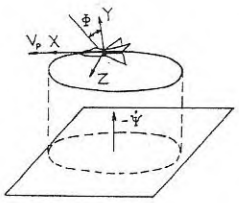
ω is vertical relative to the X'Y'Z'(earth Fixed)

$$\vec{\omega}=k'\dot{\Psi}=k_1\dot{\Psi}$$

$\therefore$In a steady level turn only ψ

Changes, θ, φ remain constant

Kinematics eq^{n}.



$$p_1 = -\dot{\Psi}_1 \sin\theta_1$$
$$q_1 = \dot{\Psi}_1 \cos\theta_1 \sin\phi_1$$
$$r_1 = \dot{\Psi}_1 \cos\theta_1 \cos\phi_1$$

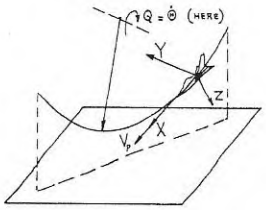
Other equation of motion remain as those of S.S flight .

Equation are used to analyze steady turning performance of A/C (CHP4)

(29-I)

- Ability to maintain (برامه‌ریزی) S.S turning flight with certain failed system
- Steady turning with all engine operating
- Steady turning with one more engine inoperative

3)Steady symmetric pull-up



X-Z plane $\Rightarrow$ $V_1=0$

Assume:

-wings level flying $p_1=\phi_1=0$

-straight $r_1=0$

$\vec{\omega}=k\dot{\theta}$

$\therefore q_1 \neq 0$ Pitch rate

(29-II)

$$\left.\begin{aligned} mw_1 q_1 &= -mg\sin\theta_1 + F_{A_{X1}} + F_{T_{X1}} \\ m(0) &= 0 + F_{A_{y1}} + F_{T_{y1}} \\ -mU_1 q_1 &= mg\cos\theta_1 + F_{A_{Z1}} + F_{T_{Z1}} \end{aligned}\right\}$$ Liner moment

$$\left.\begin{aligned} 0 &= l_{A_1} + l_{T_1} \\ 0 &= M_{A_1} + M_{T_1} \\ 0 &= N_{A_1} + N_{T_1} \end{aligned}\right\}$$ Angular moment

Kinematics eq^{n}.

$$\Psi_1 = 0, q_1 = \dot{\theta}, r_1 = 0$$

Equation are used to analyze the maneuvering capability of A/C (CHP4)

(30-I)

IN THE NAME OF GOD

Flight Dynamics, Quiz No. I, 84-85

1. What are the differences between the aircraft performance and stability & control?

2. Define static & dynamic stability and trim conditions.

3. Write down the assumptions made in deriving general equations of motions, at least 5 of them.

4.. Define each term in the following equation and what happens if ω=0, explain.

$$\frac{d\overrightarrow{V_P}}{dt} = \frac{\delta\overrightarrow{V_P}}{\delta t} + \overrightarrow{\omega} \times \overrightarrow{V_P}$$

Stability derivatives

مشتقات

$$C_L = C_{L_1} + \frac{\partial C_L}{\partial \alpha}\alpha + \frac{\partial C_L}{\partial \delta_e}\delta_e + \dots$$

$$\frac{\partial C_L}{\partial \alpha}\alpha = C_{L_\alpha}$$

$$\frac{\partial C_L}{\partial \delta_e}\delta_e = C_{L_{\delta_e}}$$

1- they are always non-Dim → آنها همیشه بدون بعد هستند

(30-II)

2-Evaluated at S.S condition → در شرایط S.S ارزیابی می‌شوند

ارزیابی می‌شوند

$$C_{D_u} = \left(\frac{\partial C_D}{\partial (u/u_1)}\right)_1$$

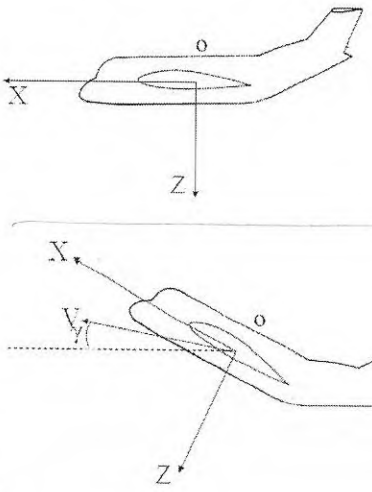
*Choice of axes

OXZ – plane of Symmetry

$$\therefore I_{XY} = I_{YZ} = 0$$

Direction of OX & Oz in the plane of symmetry are conventionally fixed in one three ways:

ترسیم



جهت Ox و Oz در صفحهٔ تقارن مرسوماً ثابت است در یکی از سه راه‌ها:

1- Principal axes

اصلی

OX & OZ coincide with the Principal axes of the a/c ∴ $I_{xz}=0$

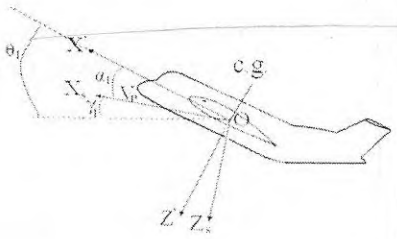
منطبق بودن

(31)

2- Stability axes

OX- points in the direction of motion

This is for symmetric S.S & straight line flight



$p_1=q_1=r_1=0$

γ_1 – flight path angle

α_1 – angle between A/C x-axis & reference

X_s – stability axis

Stability axes are body axes that remain fixed to the A/C during the motion

(32-I)

If $I_{x'x'}$, $I_{z'z'}$ and $I_{x'z'}$ are known, then

$$I_{xx} = I_{x'x'}\cos^2\alpha_1 + I_{x'z'}\sin^2\alpha_1$$

$$I_{zz} = I_{x'x'}\sin^2\alpha_1 + I_{x'z'}\sin^2\alpha_1$$

$$I_{xz} = \frac{1}{2}(I_{x'x'} - I_{x'z'})\sin(2\alpha_1)$$

(32-II)

3- body axes :

If the axes are neither principal nor stability → called body axes
x-axes is usually fixed to the longitudinal reference line in the A/C
Most convenient axes to use if aerodynamic data have been measured by balance system is a wind tunnel

(33)

اگر محورها نه اصلی باشند نه پایداری ← می‌گوییم محورهای بدنه محور x معمولاً با خط مرجع طولی در هواپیما ثابت می‌شود، اگر محورهای مناسب استفاده می‌شود که اطلاعات آیرودینامیکی به وسیله‌ی بالانس سیستم در یک تونل باد محاسبه شده‌اند.

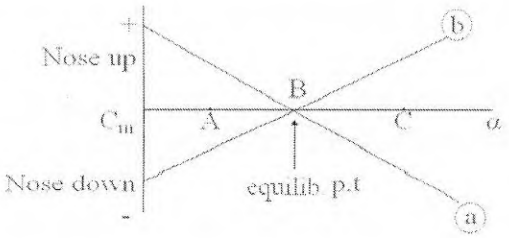
Static Stability & Control

Motion of A/C (steady & unsteady) is separated in 2 parts

1) longitudinal (symmetric) wings are level
2) lateral (asymmetric) rolling , yawing, side slipping
α and θ =constant
In static & dynamic analysis we use both motions.
Longitudinal motion is more important then lateral unless performing maneuvers
An A/C has static stability if it creates force & moments to restore to its original position when disturbed.

(34-I)

پایداری و کنترل استاتیکی:
حرکت هواپیما (پایدار و ناپایدار) در 2 قسمت جدا می‌شود.
1- طولی (متقارن) بالها هم‌تراز هستند
2- عرضی (غیرمتقارن) yawing & rolling
سرش جانبی
در آنالیز استاتیک و دینامیک ما از هر دو حرکت استفاده می‌کنیم
حرکت طولی بسیار مهم‌تر از عرضی است مگر اینکه مانورهایی انجام شود.
یک هواپیما استاتیکی پایدار است اگر آن نیرو و ممانهایی تولید کند که به موقعیت اصلی بازگرداند وقتی که آشفته شد.



pt B is trim pt $C_m=0$

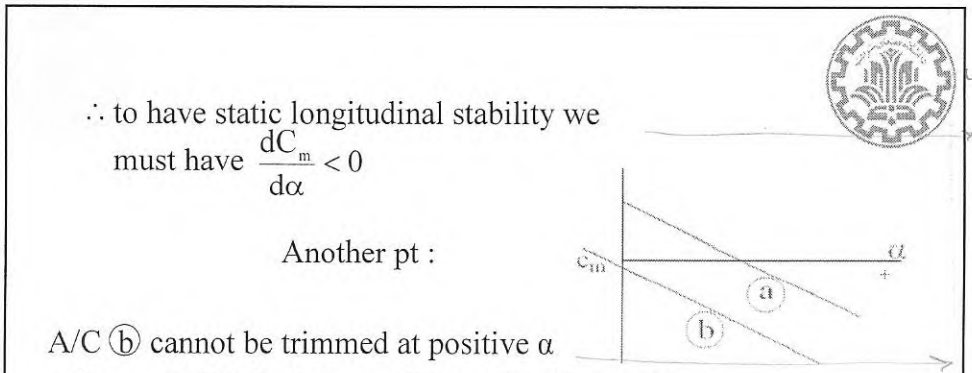
if α is increased to C , A/C ⓐ develops a nose down moment to restore to its original position while A/C ⓑ is not (nose up)

same is true when α is suddenly decreased.

A/C ⓐ generates nose-up moment.

A/C ⓑ generates nose-down moment.

(34-II)



∴ to have static longitudinal stability we must have $\frac{dC_m}{d\alpha} < 0$

Another pt :

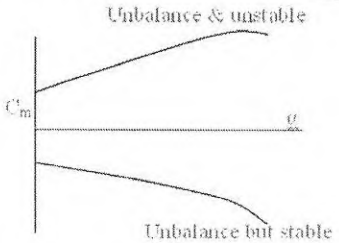
A/C ⓑ cannot be trimmed at positive α

∴ For an A/C to be in a condition of stable equilib.

$$\frac{dC_m}{d\alpha} < 0 \quad \& \quad C_{m_0} > 0$$

OR

$$C_{m_\alpha} = \frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L}\frac{dC_L}{d\alpha}$$



(35-I)

$\therefore$ For static longitudinal stability $\dfrac{dC_m}{dC_L} < 0$: برای پایداری استاتیکی طولی

Contribution of A/C components:

For total a/c , we require that $\dfrac{dC_m}{dC_L} < 0$ برای هر هواپیما نیاز داریم که $\frac{dC_m}{dC_L}$

-A/C designer want to know the contribution of wings, fuselage, tail, propulsion system etc. to the P.M & static stability charact.

-Datcom good ref. for explaining the contribution of each components.

(35-II)

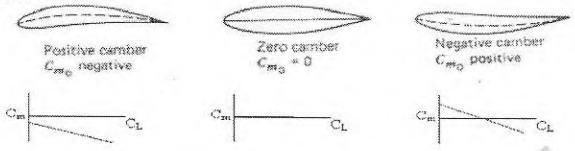
possible configurations

C_{m_α} can be made negative for any combinations of lifting surface but an aircraft must be simultaneously Balanced & Stable

A proper choice of C.G location ensures negative C_{m_α}

$\therefore$ Any configuration with positive C_{m_0} satisfies the condition for a balance & stable flight

for conventional airfoil



(36-I)

For a straight wing alone A/C (flying wing), wing camber determine the A/C charact.

Negative camber : flight possible at $\alpha>0$ & $C_L>0$

Zero camber : flight possible only at $\alpha=0$ & $C_L=0$

Positive camber : flight is not possible at any positive α or C_L

A flying wing with + camber could fly upside down without a tail provided that C.G is for ahead of mean aerodynamic center .

(36-II)

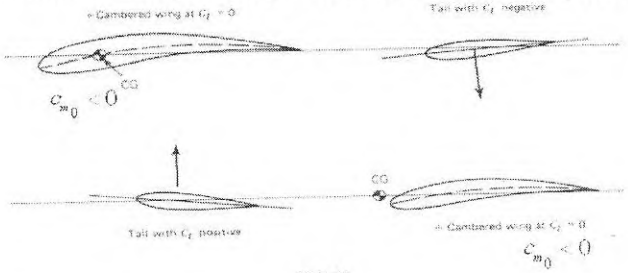
Flying wing with negative camber and straight wing are not used because

-The dynamic character tend to be unsatisfactory

-The permissible C.G rang is too small

-The drag & C_{Lmax} character are not good

Positive camber straight wing can be used with tail (forward or after)

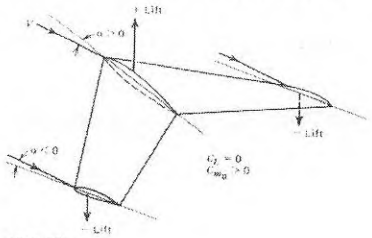


(37-I)

Auxiliary surface must produce a nose up moment when the wing is at zero lift such that $C_{m_0} > 0$

- swept back wing with twisted tips at L=0 forward part produces positive lift & the rear part negative → + couple

- For delta wing ; twist the tip employ negative camber or upturned T.E flap, reflex T.E



(37-II)

Forces & Moments

Two methods to determine them

1. Experimental
2. Theoretical/CFD/Empirical

$$F_A = F_{AW} + F_{AB} + F_{A_{H-T}} + F_{A_{V-T}}$$

Have to consider the interference effects

We use stability axis system (axis along the flight path)

$F_A = F_A(\alpha , \beta , \delta_A , \delta_e , \delta_R)$

For straight symmetrical S.S flight, the forces and moments are :

(38-I)

IN THE NAME OF GOD

Flight Mechanics II

Forces & Moments

Two methods to determine them

1. Experimental
2. Theoretical/CFD/Empirical

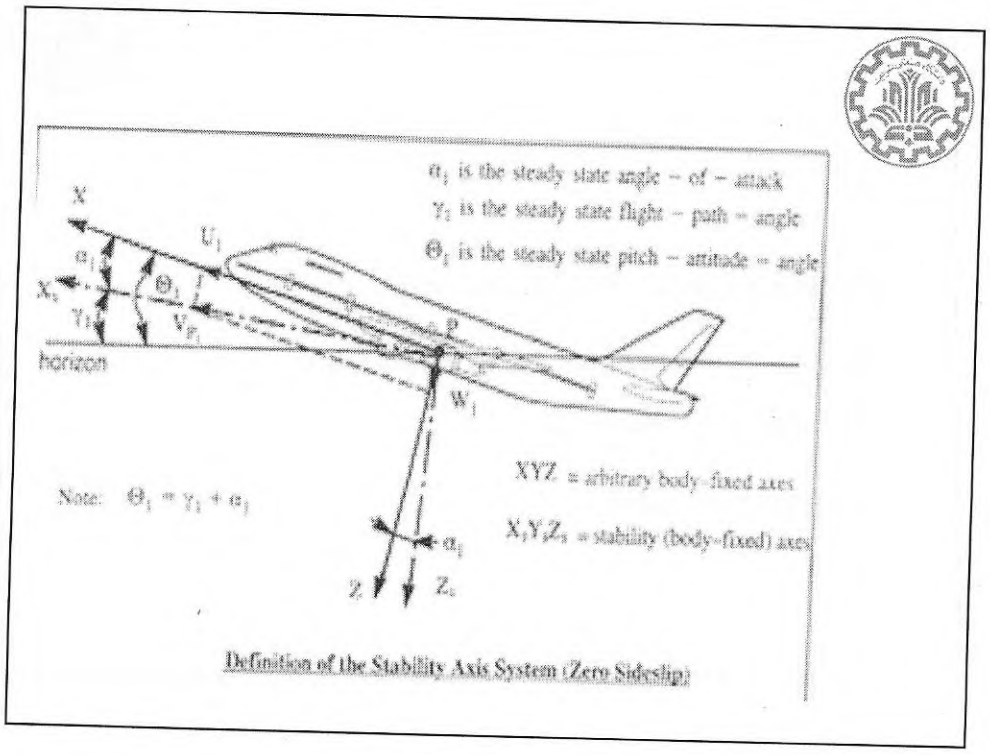
$$F_A = F_{AW} + F_{AB} + F_{A_{H-T}} + F_{A_{V-T}}$$

Have to consider the interference effects

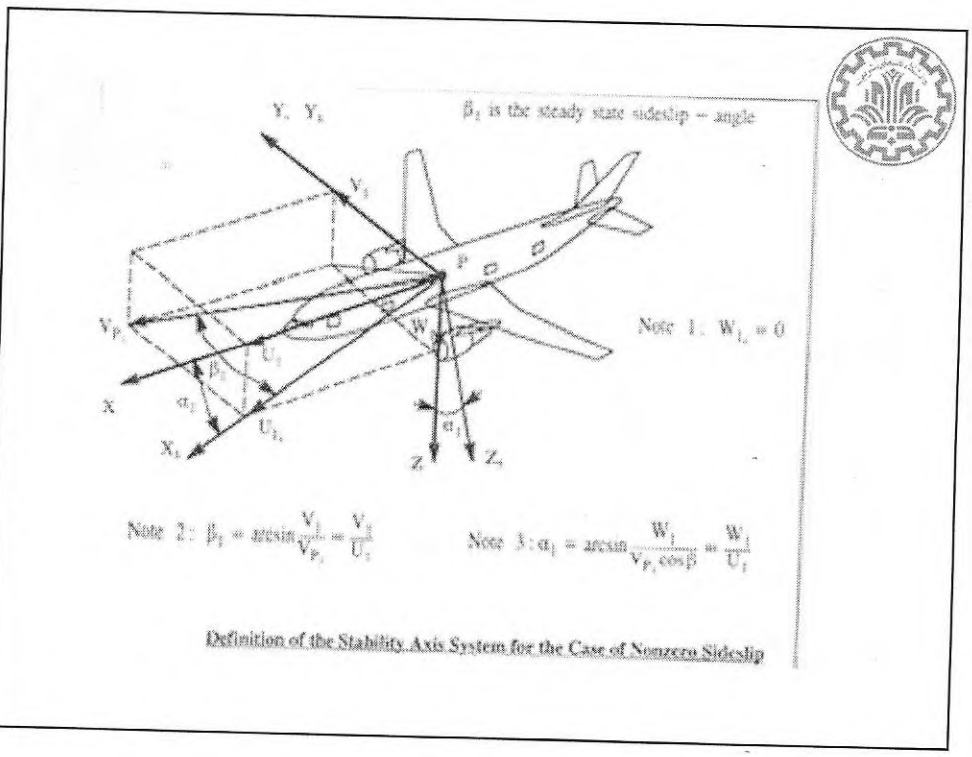
We use stability axis system (axis along the flight path)

$$F_A = F_A(\alpha, \beta, \delta_A, \delta_e, \delta_R)$$

α: sideslip angle, δ_A: Aileron angle, δ_e: elevator angle, δ_R: Rudder angle (38-I)



Definition of the Stability Axis System (Zero Sideslip)



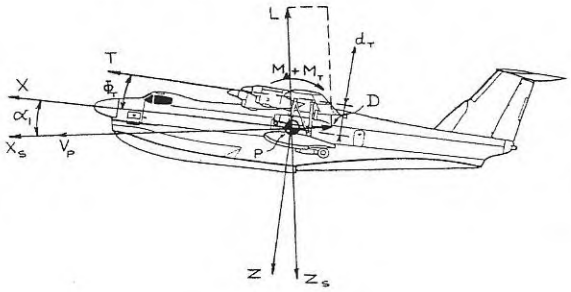
Definition of the Stability Axis System for the Case of Nonzero Sideslip

For straight symmetrical S.S flight, the forces and moments are :

$$F_{A_{X_1}} = -D \qquad F_{T_{X_1}} = T\cos(\phi_T + \alpha_1)$$

$$F_{A_{Z_1}} = -L \qquad F_{T_{Z_1}} = -T\sin(\phi_T + \alpha_1)$$

$$M_{A_1} = M_{A1} \qquad M_{T_1} = -TdT$$



(38-II)

Assume that T is known : T=T(h, M, α_p, Π)
α_p=Prop. angle of attack

-Total A/C drag :

$$F_{A_x} = -D = -\frac{1}{2}\rho V_p^2 S C_D = -\bar{q} S C_D$$

C_D=total a/c drag coefficient , C_D= f(α,δ_e,i_H,q,M,Re)
δ_e=elevator angle
i_H=stabilizer angle (incident of horizontal)

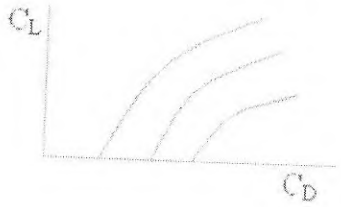
$$C_D = C_{D_0} + C_{D_\alpha}\alpha + C_{D_{iH}} i_H + C_{D_{\delta_e}} \delta_e$$

C_{D0}= Total A/C drag coefficient at α, δe, i_H=0
$C_{D\alpha}$= Total A/C drag change with angle of attack δe, i_H=0

(39-I)

C_{DiH} or $C_{D\delta e}$: Total A/C drag change with stabilizer or elevator angle ; $\alpha = 0$

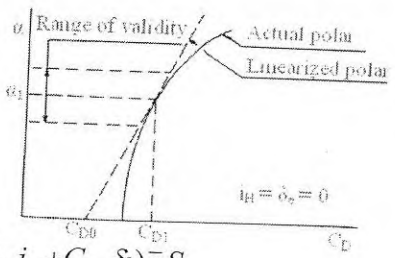
Most cases ; $C_D = \overline{C}_{D_0} + \dfrac{C_L^{\,2}}{\pi A R\, e_0}$: C_{D_0} : A/C drag at C_L=0



(39-II)

Since drag polar is quadratic, but the liner expression was reflected, the linear expression must be viewed as a local linearization.

C_{D_0} & C_{D_α} are different for each S.S. condition



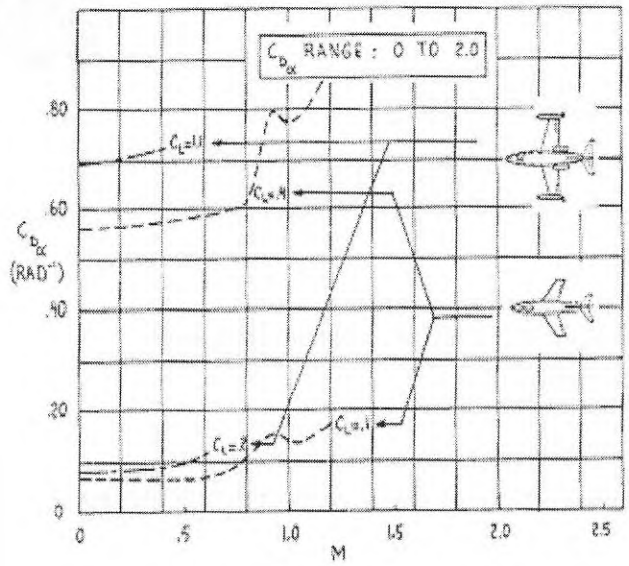
$$\therefore F_{A_{xs}} = -D = -C_D \bar{q} s = -(C_{D_0} + C_{D_\alpha}\alpha + C_{D_{i_H}} i_H + C_{D_{\delta e}} \delta e)\bar{q} S$$

$C_{D_\alpha} = f(M, \mathrm{Re}, \ldots)$, $C_{D_0} = f/s$

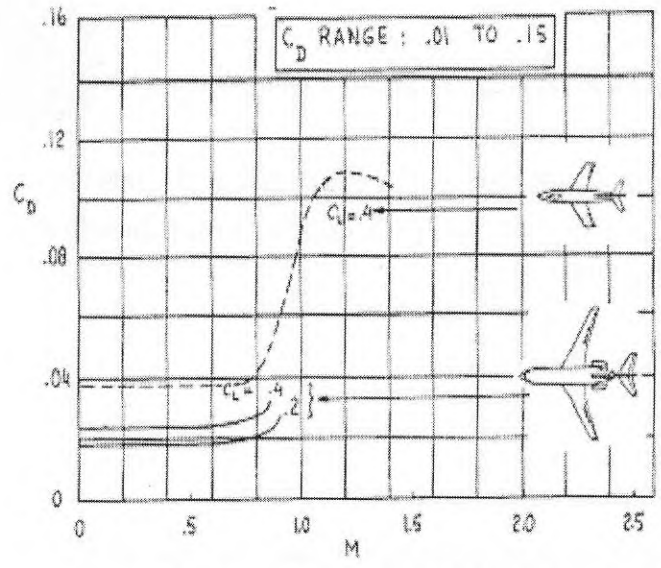
f: equivalent parasite area of A/C & f= f (s_{wett} , c_f)

(40-I)

60



Variation of C_{D_α} with Mach Number for Typical Jet Aircraft

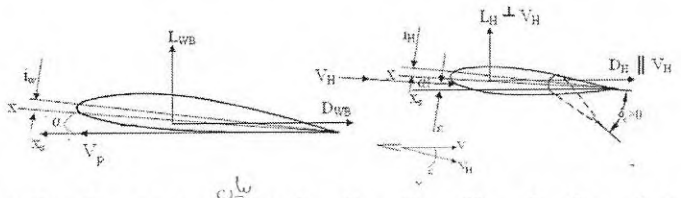


Variation of C_D with Mach Number for Typical Jet Aircraft

-Total A/C Lift

$$L = C_L \bar{q} S : C_L = C_L(\alpha, \delta_e, i_H, \bar{q}, M, \mathrm{Re}, \Pi)$$

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_H}} i_H + C_{L_{\delta e}} \delta e$$



بیان

Want to develop expressions for C_{L0}, $C_{L\alpha}$, $C_{L\delta e}$ & $C_{L_{iH}}$ in terms of A/C configuration

(40-II)

In the Z-direct

$$L = L_{WB} + L_H \cos\varepsilon - D_H \sin\varepsilon$$

واداركرن

ε: average downwash angle induced by the wing on H-T

for small ε ;cosε ≈ 1 , sinε ≈ε ≈0

$$\therefore L = L_{WB} + L_H$$

$$C_L \bar{q} s = C_{L_{WB}} \bar{q} s + C_{L_H} \bar{q}_H s_H \Rightarrow C_L = C_{L_{WB}} + \frac{q_H}{q}\frac{s_H}{s} C_{L_H}$$

Define η or η_H $= \frac{q_H}{q} = \frac{1/2\rho V_H^{\,2}}{1/2\rho V_W^{\,2}} \approx \frac{1/2\rho V_H^{\,2}}{1/2\rho V_p^{\,2}}$ "tail efficiency"

$0.8 < \eta_H < 1.2$ depending on the location of tail surface

(41-I)

• If located in the wake region of wing or fuselage or area of high downwash $q_t < q_w$ due to momentum loss in the wake

• For T-tail configuration. $\eta \approx 1$

• If tail is located in the slipstream of propeller or in the exhaust wake of a jet engine $\rightarrow q_t > q_w$

Also effect of fuselage B.L must be taken into account

η: is the measure of tail effectiveness

$$C_{L_{WB}} = C_{L_{0WB}} + C_{L_{\alpha WB}} \alpha_{WB}$$

(41-II)

$C_{L\alpha_{WB}}$ = Wing- body lift curve slops $\neq C_{L\alpha_W}$ due to interference & body lift curve slope.

If B/D ›› 1: B = wing span, D= body diameter

$\Rightarrow C_{L\alpha_{WB}} \approx C_{L\alpha_W}$ If $\frac{B}{D} \geq 6$

$\alpha_w = \alpha + i_w$; i_w = wing incidence angle

Can select x-axis S.T $i_w = 0$, but will effect $C_{L_{0\,WB}}$

i_w is determined by cruise drag, maintaining a level cabin floor in cruise & visibility on approach to landing.

(42 - I)

Horizontal tail:

$\alpha_H = \alpha_w - i_w + i_H - \varepsilon \approx \alpha + i_H - \varepsilon$

i_H: incidence of H-T for high performance A/C it is controllable from cockpit.

$$C_{L_H} = C_{L_{0_H}} + C_{L_{\alpha_H}} \alpha_H + C_{L_{\delta E}} \delta_E$$

$C_{L_{\delta E}}$: Change in total A/C lift coefficient with elevator angle

$\cong C_{L_{\alpha_H}} \tau_\varepsilon$, $\tau_\varepsilon = \partial\alpha_H / \partial\delta_E$: elevator A.O.A effectiveness

$C_{L_{0_H}} = 0$ for symmetric tail

$$\therefore C_{L_H} = C_{L_{\alpha_H}} (\alpha + i_H - \varepsilon + \tau_\varepsilon \delta_E)$$

(42 – II)

$\varepsilon = \varepsilon_0 + \dfrac{d\varepsilon}{d\alpha}\alpha$; ε_0=downwash at tail when α=0

$\varepsilon = \alpha_I \rightarrow$AERO 1

Datcom method

$$\frac{d\varepsilon}{d\alpha} = 4.44\,[K_A K_\lambda K_H (\cos(\wedge_{c/4}))^{1/2}]^{1.19}$$

K_A: AR_W , K_λ : $AR_{H\text{-}T}$, K_H: horizontal tail location factor

(43 – I)

At high subsonic speed, compressibility effect

$$\frac{\partial \varepsilon}{\partial \alpha}\Big|_M = \frac{\partial \varepsilon}{\partial \alpha}\Big|_{\text{low speed}} \cdot \frac{C_{L_\alpha}\big|_M}{(C_{L_\alpha})_{\text{low speed}}}$$

- Downwash due to flap deflection: differs for different flaps (plane, slotted)

(43 – II)

The overall lift of the A/C is:

$$C_L = C_{L_{0WB}} + C_{L_{\alpha WB}}\,\alpha + C_{L_{\alpha H}}\,\eta_H * \frac{S_H}{S}\{\alpha - (\varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha) + i_H + \tau_\varepsilon \delta_E\}$$

$$C_{L_0} = C_{L_{0WB}} - C_{L_{\alpha H}}\,\eta_H \frac{S_H}{S}\varepsilon_0 \cong C_{L_{0WB}}$$

$$C_{L_\alpha} = C_{L_{\alpha WB}} + C_{L_{\alpha H}}\,\eta_H \frac{S_H}{S}(1 - \frac{d\varepsilon}{d\alpha})$$

$$C_{L_{iH}} = C_{L_{\alpha H}}\,\eta_H \frac{S_H}{S}$$

(44- I)

$$C_{L_{\delta_E}} = C_{L_{\alpha_H}} \eta_H \frac{S_H}{S} \tau_\varepsilon$$

$$F_{A_Z} = - C_L \cdot \bar{q}.S$$

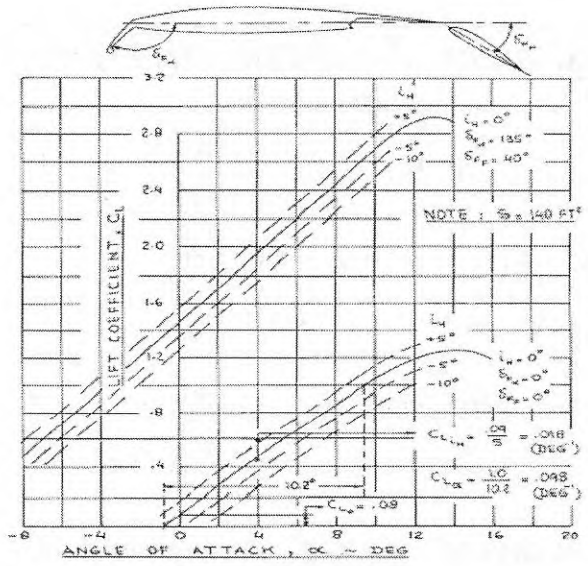
$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{iH}} i_H + C_{L_{\delta_E}} \delta_E$$

The effect of thrust on the lift is include in C_{L0}

↓

اثرات تراست در لیفت در C_{L_0} آمده است

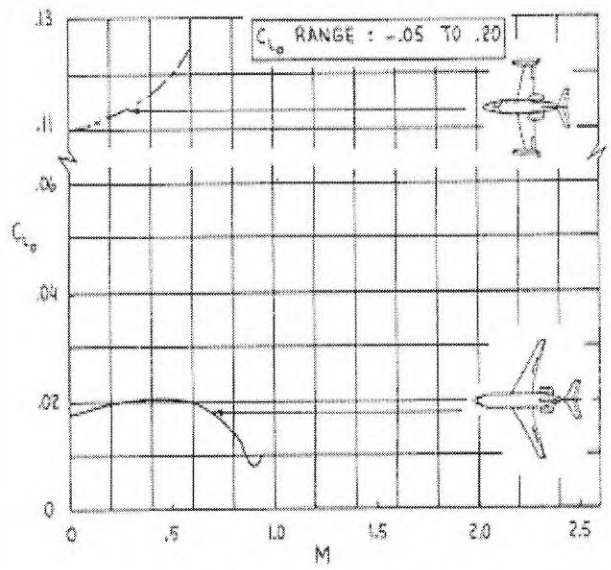
(44 – II)



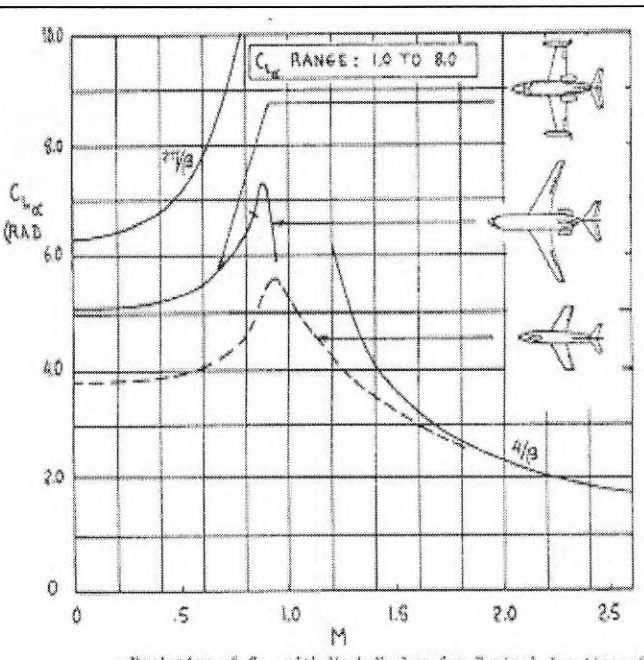
Note: Full Span Fowler Flaps, δ_{F_F} ; Full Span Krueger Flaps, δ_{F_K}

Figure 4.12 Lift Coefficient Versus Angle of Attack for a Light Airplane at Low Subsonic Speed

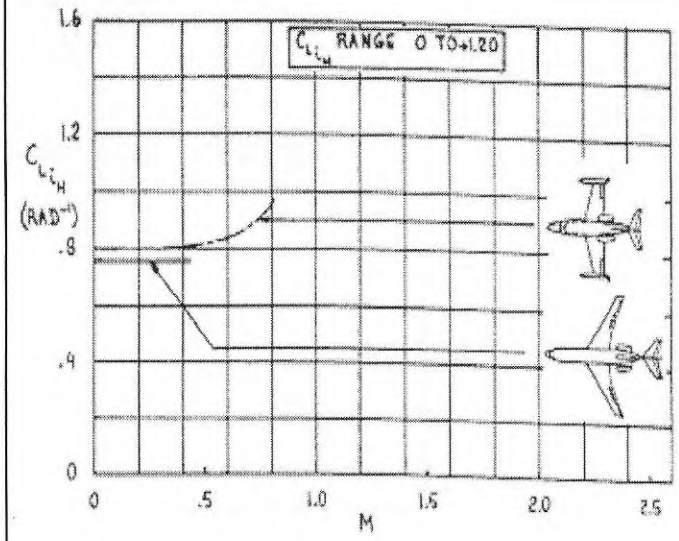
CHAPTER 4



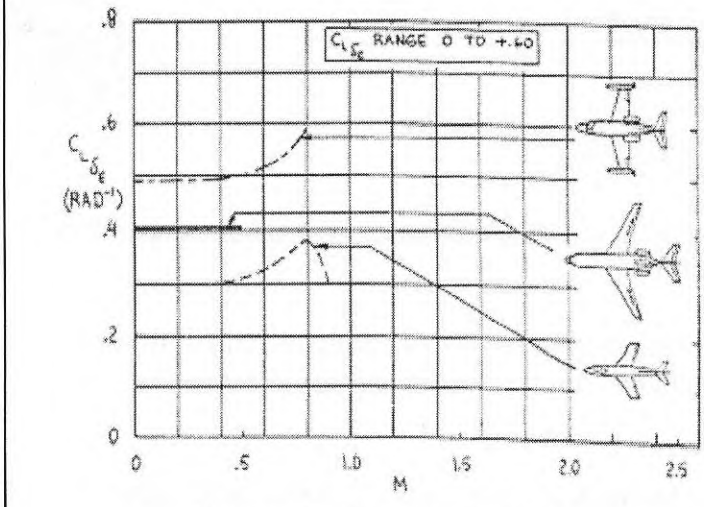
Variation of C_{l_0} with Mach Number for Typical Jet Aircraft



Variation of C_{l_α} with Mach Number for Typical Jet Aircraft



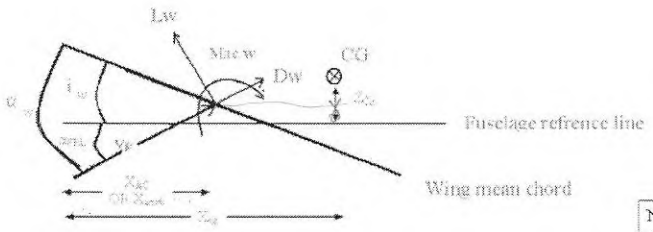
Variation of $C_{L_{i_H}}$ with Mach Number for Typical Jet Aircraft



Variation of $C_{L_{\delta_E}}$ with Mach Number for Typical Jet Aircraft

Total A/C pitching moment

A) Wing contribution



Neutral point of wing body

$\sum M_{ac_{wb}} = M_{c.g_w}$

$\sum Moments = M_{c.g_w}$

$\begin{cases} X_{ac} \text{ or } X_{ac_{wb}} \text{ or } h_{n_w} \\ X_{c.g} \text{ or } h \end{cases}$

(44 – III)

$$M_{cg_w} = L_W \cos(\alpha_w - i_w)(X_{cg} - X_{ac}) + D_w \sin(\alpha_w - i_w)(X_{cg} - X_{ac}) + L_W \sin(\alpha_w - i_w) Z_{cg} - D_W \cos(\alpha_w - i_w) Z_{cg} + M_{ac_w}$$

$$M_{cg_w} = \frac{1}{2}\rho.V^2.S\,C_{m_{cg_w}}\,\bar{C}$$

$$\Rightarrow C_{m_{cg_w}} = C_{L_w}\left(\frac{Xcg}{\bar{C}} - \frac{Xac}{\bar{C}}\right)\cos(\alpha_w - i_w) + C_{Dw}\sin(\alpha_w - i_w)\left(\frac{Xcg}{\bar{C}} - \frac{Xac}{\bar{C}}\right) + C_{L_w}\sin(\alpha_w - i_w)\frac{Zcg}{\bar{C}} - C_{D_W}\cos(\alpha_w - i_w)\frac{Zcg}{\bar{C}} + C_{m_{ac_w}}$$

(45 - I)

To find variation of $C_{m_{cg_w}}$ with α_w must plot the above equation.

برای پیدا کردن متغیر $C_{m_{cg_w}}$ با α_w باید نمودار معادله بالا را رسم کرد

For small α

$\Rightarrow \text{Cos}(\alpha_w - i_w) \approx 1$; $\sin(\alpha_w - i_w) \approx (\alpha_w - i_w)$

& $C_L \gg C_D$

i.e for high $\alpha \approx 15°$, and high value of drag $\approx L/D \approx 6$

$\Rightarrow$ Ratio of 2nd term to 1st will be about 1/ 23

- Also, assume that the vertical contribution is negligible.

$$C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w}\left(\frac{x_{cg}}{\bar{C}} - \frac{x_{ac}}{\bar{C}}\right)$$

$$= C_{m_{ac_w}} + \left(C_{L_{0_w}} + C_{L_{\alpha_w}}\alpha_w\right)\left(\frac{x_{cg}}{\bar{C}} - \frac{x_{ac}}{\bar{C}}\right)$$

(45 – II)

$$\Rightarrow C_{m_{cg_w}} = C_{m_{0W}} + C_{m_{\alpha_w}}\alpha_w$$

$$\Rightarrow C_{m_{0W}} = C_{m_{ac_w}} + C_{L_{0W}}\left(\frac{Xcg}{\bar{C}} - \frac{Xac}{\bar{C}}\right)$$

$$\Rightarrow C_{m_{\alpha w}} = C_{L\alpha w}\left(\frac{Xcg}{\bar{C}} - \frac{Xac}{\bar{C}}\right)$$

For a statically stable wing $\frac{dCm}{d\alpha} < 0$

Aerodynamic center must be lie after of C.G

مرکز آیرودینامیکی باید بعد از CG باشد.

Also $Cm_0 > 0$ to trim the A/C at $\alpha > 0$

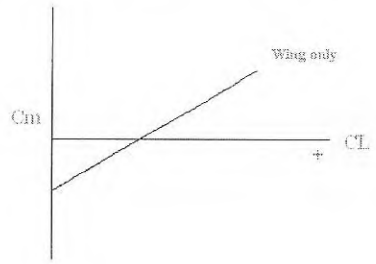
$\therefore$ The only way is to have negative camber

Airfoil or an airfoil which has a reflexed T.E

تنها راه داشتن یک ایرفویل camber منفی است یا یک ایرفویل که T.E reflex داشته باشد

(46 – I)

For most conventional A/C, the wing contribution to static longitudinal stability is usually destabilizing.

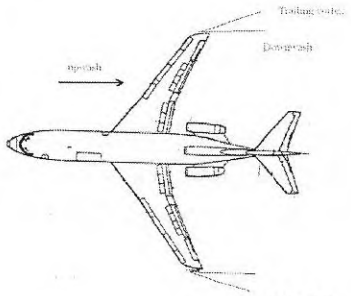


(46 – II)

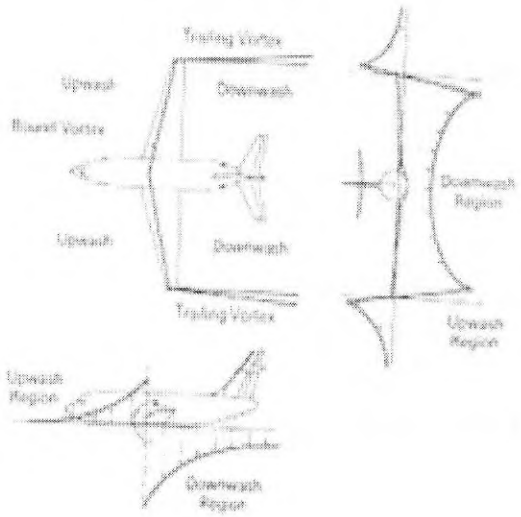
B) Tail contribution

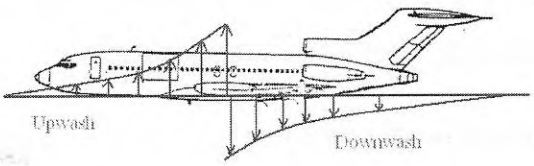
Tail surface can be located either forward or aft.of the C.G. both are influenced by the flow filed created by the wing.

Canard is effected by the upwash flow whereas the aft. Tail is subjected the downwash one.



(47- I)

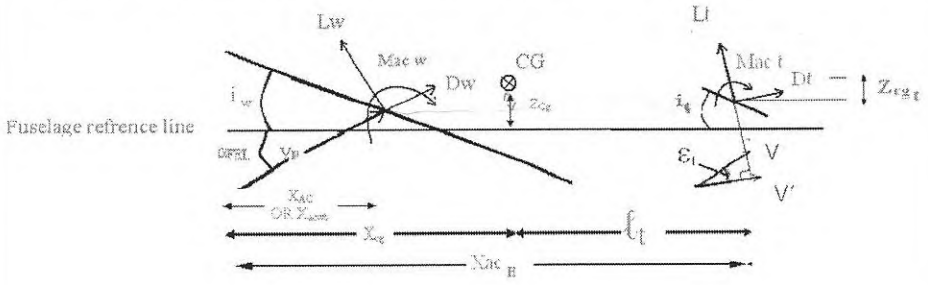




بزرگی

Magnitude of the upwash & downwash depends on the location of the tail surface w.r.t the wing.

بزرگی upwash و downwash به مکان سطح دم نسبت به بال بستگی دارد.



(47 – II)

$$M_t = -\ell_t \left[L_t \cos(\alpha_{FRL}-\varepsilon) + D_t \sin(\alpha_{FRL}-\varepsilon) \right]$$
$$- Z_{cg_t} \left[D_t \cos(\alpha_{FRL}-\varepsilon) - L_t \sin(\alpha_{FRL}-\varepsilon) \right] + M_{ac_t}$$

Experience has shown that for most A/C the 1st term is the dominating one

آزمایشات نشان می‌دهد که برای بیشتر هواپیماها ترم اول بر آن یکی غلبه می‌کند.

also $L_t \gg D_t$

$\Rightarrow M_t = -\ell_t L_t$ for small $(\alpha_{FRL}-\varepsilon)$

; $M_t = q_t . \ell_t . S_t . C_{L_t}$

$$C_{m_t} = \frac{M_t}{\frac{1}{2}\rho V^2 S\bar{C}} = -\frac{\ell_t}{S\bar{C}} \frac{S_t}{q} \frac{q_t}{q} . C_{L_t}$$

(48 – I)

$$V_H = \frac{\ell_t . S_t}{S\bar{C}}$$ horizontal tail volume ratio or tail volume

$$= C_{m_t} = - V_H \eta_H C_{L_t}$$

$$\alpha_t = \alpha_w - i_w + i_t - \varepsilon$$

$$C_{L_t} = C_{L\alpha_t} \alpha_t = C_{L\alpha_t} (\alpha_{wb} - \varepsilon + i_t)$$

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha$$

(48 – II)

$$C_{m_{cg_t}} = - V_H \eta_H C_{L\alpha_t} (\alpha_{wb} - \varepsilon_0 - \frac{d\varepsilon}{d\alpha} \alpha_w + i_t)$$

$$C_{m_{cg_t}} = V_H \eta_H C_{L\alpha_t} (\varepsilon_0 - i_t) - V_H \eta_H C_{L\alpha_t} (1 - \frac{d\varepsilon}{d\alpha}) \alpha_w$$

$$C_{m_{cg_t}} = C_{m_0} + C_{m_\alpha} \alpha$$

$$C_{m_{0t}} = V_H \eta_H C_{L\alpha_t} (\varepsilon_0 - i_t)$$

$$C_{m_{\alpha_t}} = - V_H \eta_H C_{L\alpha_t} (1 - \frac{d\varepsilon}{d\alpha})$$

(48 – III)

∴ Though the wing contribution to C_{m_0} was negative, the tail contribution to C_{m_0} can be used to ensure the $(C_{m_0})_{A/C} > 0$

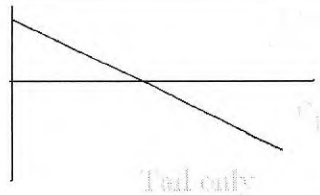
To do that adjust i_t and to $\uparrow C_{m_0}$ due to tail, should mount the tail plane at a negative angle of incidence to the fuselage.

For $C_{m_{\alpha_t}} < 0$, adjust $V_H, C_{L_{\alpha_t}}$

Contribution of $C_{m_{\alpha_t}}$ will be more neg. if $\uparrow l_t$ or S_t or $C_{L_{\alpha_t}}$ by $\uparrow AR_t$.

∴ A tail surface located aft. of the wing can be used to ensure that the A/C has a positive C_{m_0} and a negative C_{m_α}.

(49 -I)



Forward tail (canard)

+ Relatively free from wing or propulsive flow interference.
+ More attractive for trimming the large nose-down moment produced by high lift devices.

To contract the nose-down moment, canard must produce .+ lift(↑ overall lift), while aft. wing tail should produce .– lift (↓ the A/C overall lift)

(49 - II)

- Canard must always be mounted fwd. of C.G. to maintain trim.
- Canard always has a destabilizing effect on the A/C (but by proper location of C.G. can make A/C stable) . (Disad.)
- Aerodynamic center of the main wing must always be aft. of the C.G. for a stable wing - canard configuration.

It is the main wing and not the canard that provides stability for the wing- canard configuration.

To maintain both trim and static stability, C.G .for any wing-canard A/C must be located between the wing and the canard.

(49-B- I)

- In a trimmed flight both the wing and the canard will always carry positive lift.

+ Canard provides added maneuverability, Spin resistance, and may interact favorably with the main wing to $\uparrow C_{L_{max}}$

A/c's which has canard," Wright brothers", beech starship, voyager,...

− Reduces efficiency of the wing due to its downwash. (Disad.)

(49-B -II)

Fuselage contribution

- Body and nacelles contributions are complex.

Optimum design of fuselage at min. drag is a body $\frac{L}{D} << 1$

Mostly circular cross sections are used

(50 - I)

Munk showed that (for pitching charact. of airships neglect viscosity and used ideal fluid).

$$\frac{dM}{d\alpha} = f\left(Vol., \frac{1}{2}\rho V^2\right)$$

Using energy and momentum relationship

↓ Body vol. ↓ Dynamic pressure

Muthopp extended Munk's idea to analyze C_{m_0} and C_{m_α}

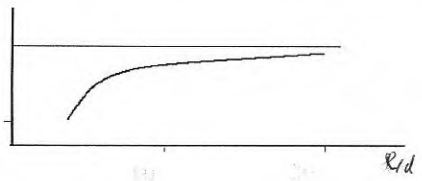
$$C_{m_{0f}} = \frac{K_2 - K_1}{36.5 S \bar{C}} \int_0^{l_f} W_f^2 \left(\alpha_{0_w} + i_f\right) dx$$

(50- II)

This can be approximated as

$$C_{m_{0f}} = \frac{K_2 - K_1}{36.5 S \bar{C}} \sum_{x=0}^{x=l_t} W_f^2 \left(\alpha_{0_w} + i_f\right) \Delta X$$

K_2- K_1: correction factor for body fineness ratio



S : wing ref. area

$\bar{C}$: wing mean aerodynamic chord

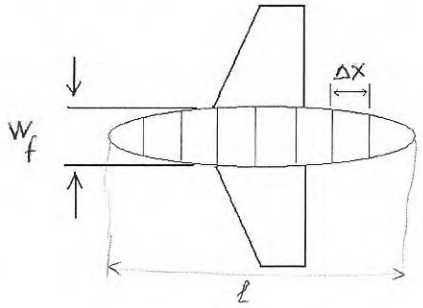
W_f : Avg .width of the fuselage section

α_{0_w} : wing zero – lift angle relative to the fuselage reference line

(51-I)

i_f : incidence of the fuselage camber line relative to the fuselage reference line at the center of each fuselage increment.

ΔX : length of the fuselage increment



(51- II)

Due to upwash and downwash effect

$$C_{m\alpha_f} = \frac{1}{36.5 S \bar{C}} \int_0^{l_f} W_f^2 \frac{\partial \varepsilon}{\partial \alpha} dx$$

$$= \frac{1}{36.5 S \bar{C}} \sum_{x=0}^{x=l_f} W_f^2 \frac{\partial \varepsilon_u}{\partial \alpha} \Delta x \qquad 1/\text{deg}$$

$\frac{\partial \varepsilon}{\partial \alpha}$ - Change in local flow angle with α

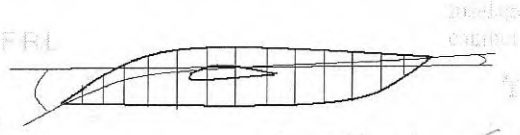
Fig. 3.33 of book

(51- III)

Can divide the fuselage into segments and the local A.O.A of each section.

$\alpha_{L_0} = \alpha_g + i_f$ →local induced α

i_f : due to wing upwash or downwash



for portion of fuselage ahead of wing $\partial\varepsilon_u / \partial\alpha > 1 \longrightarrow$ local A.O.A ↑.

Behind the wing, due to the wing downwash, local α ↓

(52 -I)

$d\varepsilon/d\alpha$ varies linearly from zero to (1- $d\varepsilon/d\alpha$) at the tail.

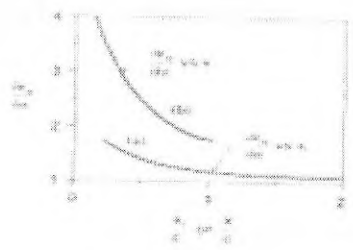
The region between the wing L.E and its T.E $\partial\varepsilon/\partial\alpha \approx 0$

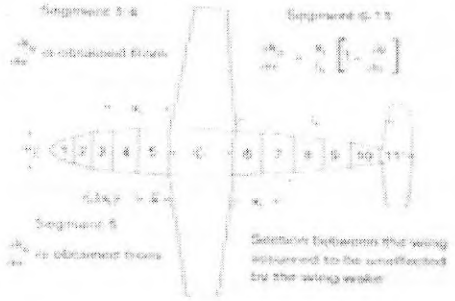
∴ The fuselage contribution is generally unstable creates nose-up pitching moment with ↑ C_L .Major problem comes from the portion ahead of the wing

∴ Long noses should be avoided.

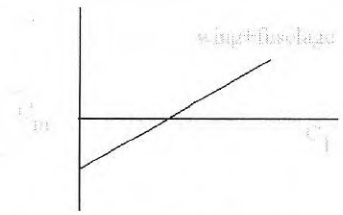
Nacelles act similar to a fuselage and may add considerable unstable P.M. when located ahead of C.G.

(52 -II)



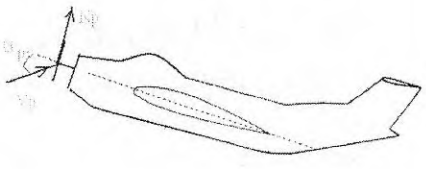


96



Power effect

Propulsion unit has a significant effect on both longitudinal trim and static stability of the A/C. It's effect depends on location of thrust line w.r.t vertical location of the C.G.

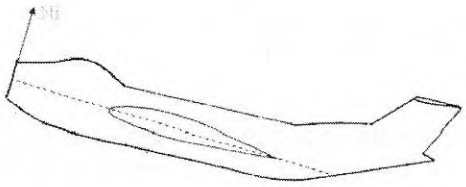


Creates a P.M about C.G

(53- I)

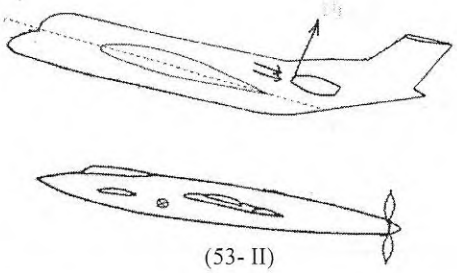
واحد موتور یک اثر مناسب هم بر trim طولی و هم بر پایداری استاتیکی هواپیما دارد. این اثر به مکان خط تراست نسبت به مکان عمودی CG بستگی دارد

یک P.M حول CG تولید می کند



Propulsive effects on A/C stability are commonly estimated from powered tunnel models.

اثرات راستی در پایداری هواپیما معمولاً با نیروی مدلهای تونل تخمین زده می‌شود



(53- II)

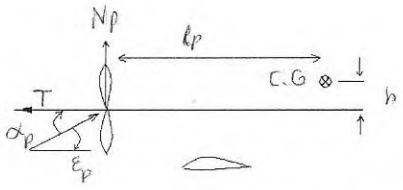
Propeller A/C

The effect is divided in to 2 groups:

اثرات به دو گروه تقسیم می‌شود.

- Direct propulsion contribution

اثرات مستقیم پیشران



↓

up wash due to the wing

$$M_{cg_p} = T \times h + N_p \times l_p$$

(54- I)

$$C_{m_p} = T_c \frac{2D^2}{S_w}\frac{h}{c}N + C_{N_p}\frac{l_p}{S_w}\frac{S_p}{c}N$$

N: No. of propellers

$S_p = \frac{\pi D^2}{4}$ Propeller disc area

$C_{N_p} = \frac{N_p}{qS_p}$ Tc: thrust coeff. $= \frac{T}{\rho V^2 D^2}$

$$\frac{dC_{m_p}}{dC_L} = \frac{dT_c}{dC_L} * \frac{2D^2}{S_w}\frac{h}{c}N + \frac{dC_{N_p}}{dC_L}\frac{l_p}{S_w}\frac{S_p}{c}N$$

↓

Must be determine for each particular case

≈ 0.5 for a single propeller

(54 -II)

For fighter a/c $\frac{D^2}{s_w} \approx 0.25$

$$\Rightarrow \left(\frac{dC_{mp}}{dC_L}\right)_T \approx 0.25\frac{h}{c}$$

A high thrust line will have a stable effect while the low one will have an destabilizing one

$\left(\frac{dC_m}{dC_L}\right)_{N_p} \approx 0.02\frac{l_p}{c}$ For typical single engine A/C

If the propeller is located for ahead of c.g $(\frac{l_p}{c} > 0) \Rightarrow$ it will destabilize the A/C. while for a pusher a/c it will stabilize it.

(55-I)

slipstream اثرات متقابل و انفعال جریان تولید شده در پشت ملخ

Effects of slipstream interaction

a) Effect of slipstream on wing-fuselage moment
b) Effect of slipstream on lift coefficient
c) Effect of slipstream on downwash at the H-T
d) Effect of increased slipstream dynamic pressure on the tail

the first two effects are negligible, while the second two are important.

Jet effects

Jet Analysis is much simpler than propeller one due to complicated effect of the propellers slipstream effect on H-T.
Jet blast is kept at a far distance from the tail due to its heat.

(55-II)

3 major effects

Direct thrust, direct normal force effects at the air-duct inlet & the effect of induced flow at the tail due to the inflow to jet blast.

Rocket installation encounters only first & last

i) Direct thrust $C_m = \dfrac{T \times Z_t}{q S_w C}$

$$q = \frac{W/S_w}{C_L} \Rightarrow C_m = \frac{T \times Z_t \times C_L}{W^* C}$$

$$\left(\frac{dC_m}{dC_L}\right)_{th} = \frac{T \times Z_t}{w \times c}$$

If $Z_t > 0 \Leftrightarrow$ destabilizing effect

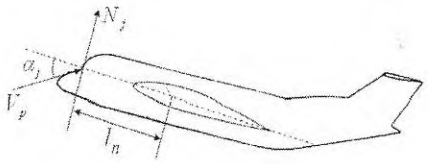


(56-I)

ii) Normal force effect

for small α

$$N_j = \frac{W_a}{g} V_p \frac{\alpha_j}{57.3}$$



N_j: normal force

W_a: weight of air lb/sec

α_j: Angle between

(56-II)

$\alpha_j = \alpha \frac{d\beta}{d\alpha} \mapsto$ upwash/downwash effect

To a first approximation (تخمینی)

$$\left(\frac{dC_m}{dC_L}\right)_j = \frac{W_0}{g} \frac{d\beta / d\alpha}{\alpha_w} \frac{l_n}{C} \frac{0.05}{S_w V_p}$$

iii) Third contribution

only for cases where jet blast (مربان هوا) passes under or over the H-T surface must be obtained from literature (توسط نتایج تجربیات)

فقط مواردی که جریان هوای جت از زیر یا روی سطح H-T عبور کند باید با توجه به نتایج تجربی نشان داده شود.

mostly the effects are as :

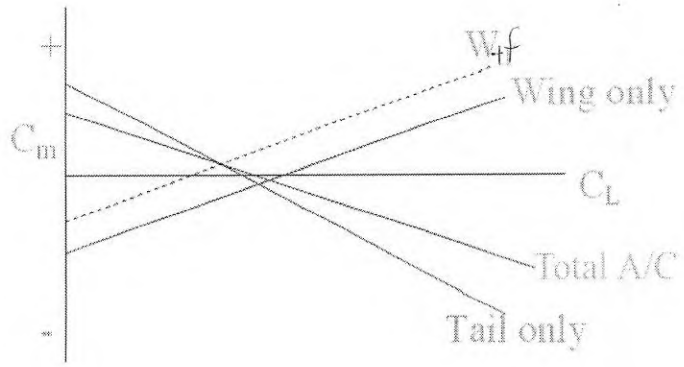
normal force $\Delta \frac{dC_m}{dC_L} \approx 0.01 - 0.03$

(57-I)

flow inclination (تمايل - انحراف)

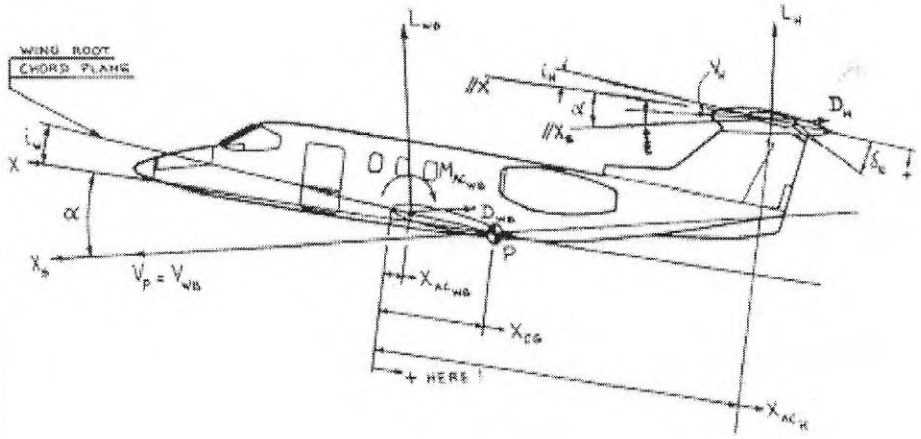
- jet under the tail $\Delta\frac{dC_m}{dC_L} \approx 0.04$
- jet behind the tail $\Delta\frac{dC_m}{dC_L} \approx 0$

(57-II)



(58-I)

Total aircraft pitching moment



$$M_A = C_m \bar{q} S \bar{c}$$

$$M_A = C_{m_{ac_w}} \bar{q} S \bar{c} + L_{WB}(x_{cg} - x_{ac_{WB}})\cos\alpha + D_{WB}(x_{cg} - x_{ac_{WB}})\sin\alpha$$

$$- L_H(x_{ac_H} - x_{cg})\cos(\alpha - \varepsilon) - D_H(x_{ac_H} - x_{cg})\sin(\alpha - \varepsilon) + C_{mac_H}$$

+ terms that depend on z

Assumption :
α is very small → cos α≈1 , sinα≈0

Divide by qSc : let $\bar{x} = \frac{x}{\bar{c}}$

$$\Rightarrow C_m = C_{m_{ac_{WB}}} + C_{L_{WB}}(\bar{x}_{cg} - \bar{x}_{ac_{WB}}) - C_{L_H}\frac{\bar{q}_H}{\bar{q}}\frac{s_H}{s}(\bar{x}_{ac_H} - \bar{x}_{cg})$$

Define $\bar{V}_H \equiv \frac{s_H}{S}(\bar{x}_{ac_H} - \bar{x}_{cg})$ Horizontal tail vol. coeff.

Plays an important role in the process of initial 'sizing' of the H-T of new a/c designs.

$$C_{L_H} = C_{L_{\alpha H}}(i_H + \alpha - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + \tau_e\delta_e)$$

$$C_{L_{WB}} = C_{L_{0_{WB}}} + C_{L_{\alpha_{WB}}}\alpha$$

$$\Rightarrow C_m = (C_{L_{0_{WB}}} + C_{L_{\alpha_{WB}}}\alpha)(\bar{x}_{cg} - \bar{x}_{ac_{WB}}) + C_{m_{ac_{WB}}}$$
$$- C_{L_{\alpha_H}}\eta_H\bar{V}_H(i_H + \alpha - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha + \tau_e\delta_e)$$

(59-I)

The $\bar{x}_{ac_{WB}}$ is usually expressed as : $\bar{x}_{ac_{WB}} \approx \bar{x}_{ac_W} + \Delta\bar{x}_{ac_B}$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{iH}}i_H + C_{m_{\delta e}}\delta_e$$

Some books

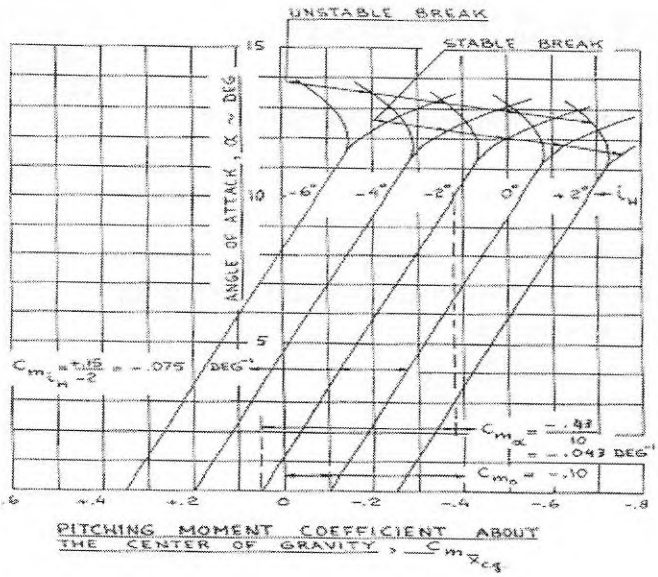
$$\Rightarrow C_{m_0} = C_{L_{0_{WB}}}(\bar{x}_{cg} - \bar{x}_{ac_{WB}}) + C_{L_{\alpha_H}}\eta_H\bar{V}_H\varepsilon_0 + C_{m_{ac_{WB}}} + C_{m_{0_f}}, \quad \varepsilon_0 \approx 0$$

Some books

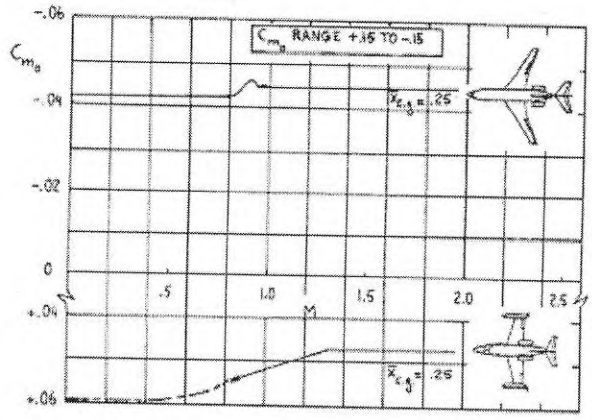
$$C_{m_\alpha} = C_{L_{\alpha_{WB}}}(\bar{x}_{cg} - \bar{x}_{ac_{WB}}) - C_{L_{\alpha_H}}\eta_H\bar{V}_H(1 - \frac{d\varepsilon}{d\alpha}) + C_{m_{\alpha_f}}$$

$$C_{m_{iH}} = -C_{L_{\alpha_H}}\eta_H\bar{V}_H$$
$$C_{m_{\delta e}} = -C_{L_{\alpha_H}}\eta_H\bar{V}_H\tau_e$$
$$\Rightarrow$$ Longitudinal control Power derivatives

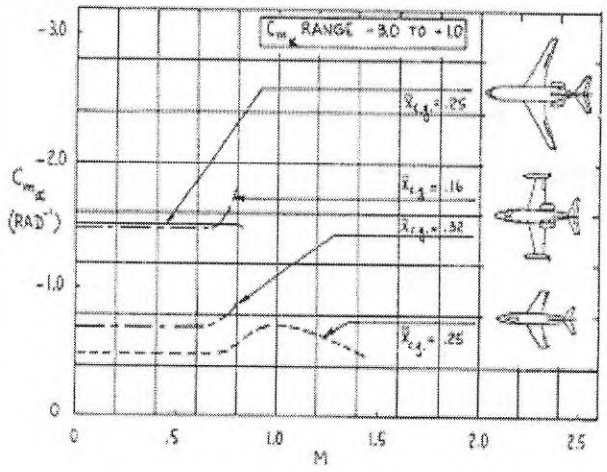
(59-II)



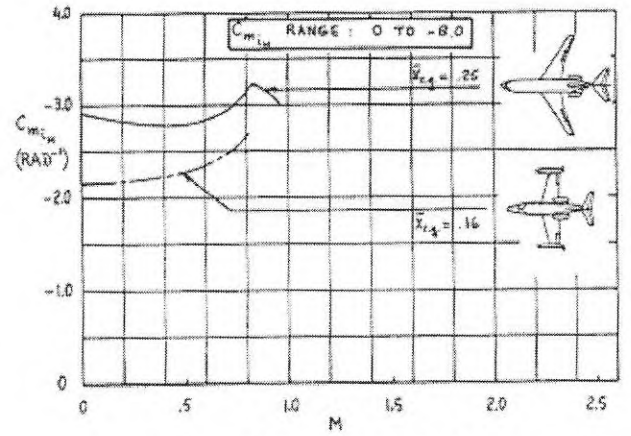
Typical Example of the Pitching Moment Slope of an Airplane



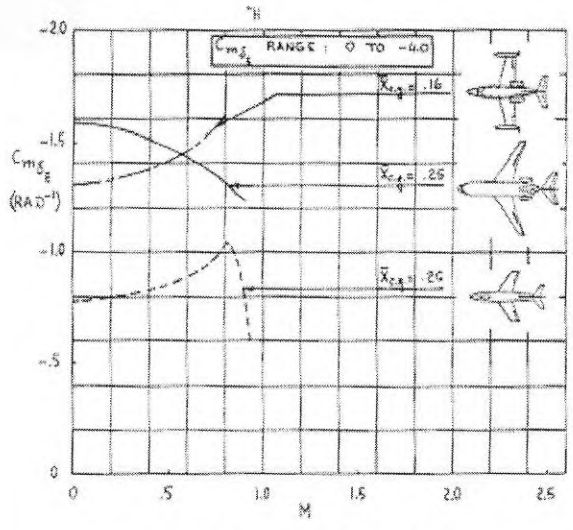
Variation of C_{m_o} with Mach Number for Typical Jet Aircraft



Variation of C_{m_α} with Mach Number for Typical Jet Aircraft



Variation of $C_{m_{i_H}}$ with Mach Number for Typical Jet Aircraft



Variation of $C_{m_{\delta_E}}$ with Mach Number for Typical Jet Aircraft

Define the a/c aerodynamic center " the point about which the moment coefficient is independent of AOA ", $X_{a.c}$

مرکز آیرودینامیکی هواپیما نقطه‌ای است که ضریب ممان به زاویه حمله وابسته نباشد X_{ac}

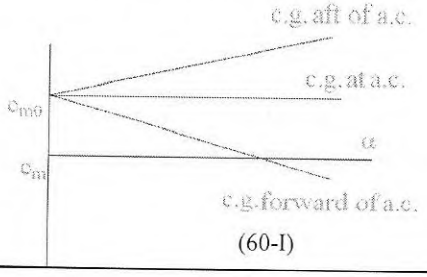
$C_{m\alpha}$: Static longitudinal stability derivative ; important to A/C stability & control

$C_{m\alpha}$: مشتق پایداری طولی استاتیکی برای کنترل و پایداری هواپیما مهم است

$$C_{m_\alpha} = 0 \Rightarrow \bar{x}_{cg} = \bar{x}_{ac}$$ (Stick-fixed neutral point, or neutral point)

This is to insure that the a/c possess static longitudinal stability

این می‌تواند ضمانت کند که هواپیما پایداری طولی استاتیکی داشته باشد.



(60-I)

$$C_{m_\alpha}=0\Rightarrow C_{L_{\alpha_{WB}}}(\bar{X}_{ac}-\bar{X}_{ac_{WB}})-C_{L_{\alpha_H}}\eta_H V_H\left(1-\frac{d\varepsilon}{d\alpha}\right)=0$$

$$\Rightarrow \bar{X}_{ac}=\frac{C_{L_{\alpha_{WB}}}\bar{X}_{ac_{WB}}+C_{L_{\alpha_H}}\eta_H\frac{S_H}{S}\bar{X}_{ac_H}\left(1-\frac{d\varepsilon}{d\alpha}\right)}{C_{L_{\alpha_{WB}}}+C_{L_{\alpha_H}}\eta_H\frac{S_H}{S}\left(1-\frac{d\varepsilon}{d\alpha}\right)}$$

$$\bar{X}_{ac}=\frac{\bar{X}_{ac_{WB}}+\frac{C_{L_{\alpha_H}}}{C_{L_{\alpha_{WB}}}}\eta_H\frac{S_H}{S}\bar{X}_{ac_H}\left(1-\frac{d\varepsilon}{d\alpha}\right)}{1+\frac{C_{L_{\alpha_H}}}{C_{L_{\alpha_{WB}}}}\eta_H\frac{S_H}{S}\left(1-\frac{d\varepsilon}{d\alpha}\right)}$$ " Stick fixed N.P "

Before we had $C_{L_\alpha}=C_{L_{\alpha_{WB}}}+C_{L_{\alpha_H}}\eta_H\frac{S_H}{S}\left(1-\frac{d\varepsilon}{d\alpha}\right)$

(60-II)

Static margin

$$\Rightarrow C_{m_\alpha}=[C_{L_{\alpha_{WB}}}+C_{L_{\alpha_H}}\eta_H\frac{S_H}{S}\left(1-\frac{d\varepsilon}{d\alpha}\right)]\bar{X}_{cg}-[C_{L_{\alpha_{WB}}}\bar{X}_{ac_{WB}}+C_{L_{\alpha_H}}\eta_H\frac{S_H}{S}\bar{X}_{ac_H}\left(1-\frac{d\varepsilon}{d\alpha}\right)]$$

$$C_{m_\alpha}=C_{L_\alpha}\bar{X}_{cg}-C_{L_\alpha}\bar{X}_{ac}=C_{L_\alpha}(\bar{X}_{cg}-\bar{X}_{ac})$$

معادلات شامل اثرات P.M ناشی از پیشرانش نیست.

Note:

1-Aove equations do not include the P.M contribution due to the propulsive installation.

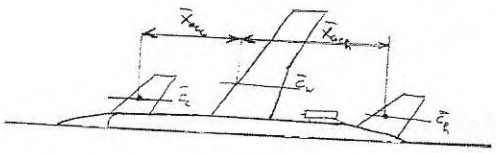
For propeller-driven airplane significant shift in a.c. due to propeller normal force and propeller tilt angle.

2-Equation applies to tail-aft airplane only, for canard and three surface airplane (Beech starship 1, Piaggio P-80) For airplane that canard does not significantly interfere with the wing (or tail) flow field, for a 3 surface A/C

برای عمده هواپیماهای ملخی ... خاصیت برروی عمودی ملخ ولخ شدن زاویه ملخ جابجایی a.c دارد

معادلات فقط برای هواپیمایی که دم عقب است عمل می‌کند.

$$\bar{X}_{ac_{AIC}} = \frac{\bar{X}_{ac_{wf}} - \frac{C_{L_{\alpha_c}}}{C_{L_{\alpha_{wf}}}}\eta_c \frac{Sc}{S}\bar{X}_{ac_c}(1+\frac{d\varepsilon_c}{d\alpha}) + \frac{C_{L_{\alpha_h}}}{C_{L_{\alpha_{wf}}}}\eta_h \frac{S_h}{S}\bar{X}_{ac_h}(1-\frac{d\varepsilon}{d\alpha})}{1+\frac{C_{L_{\alpha_c}}}{C_{L_{\alpha_{wf}}}}\eta_c \frac{S_c}{S}(1+\frac{d\varepsilon_c}{d\alpha}) + \frac{C_{L_{\alpha_h}}}{C_{L_{\alpha_{wf}}}}\eta_h \frac{S_h}{S}(1-\frac{d\varepsilon}{d\alpha})}$$



$$\frac{dC_m}{dC_L} = -S.M$$ Or stick fixed static margin

این میزان پایداری استاتیکی هواپیما نسبت به اغتشاشات فاصله میان CG و AC هواپیماست برای اینکه $\frac{\partial C_m}{\partial \alpha} < 0$ باید CG جلوی neutral point باشد

It is a measure of the static stability of the airplane with respect to disturbances. It is a distance between the C.G and the airplane a.c. To have $\frac{\partial C_m}{\partial \alpha} < 0$ we must have C.G forward of the neutral point.

Putting a tail on airplane will: → نداشتن یک دم (در هواپیما دارد:

1-provide positive C_{m_0} مثبت فراهم می‌آورد C_{m_0}

$$C_{m_0} = C_{L_{O_{WB}}}(\bar{X}_{cg} - \bar{X}_{ac_{wB}}) + C_{L_{\alpha_{HT}}}\eta_H \bar{V}_H \varepsilon_0 + C_{m_{0_f}} + C_{mac_{wB}}$$

2-control of neutral point location, by appropriate selection of V_H → کنترل مکان N.P به وسیله انتخاب مناسب V_H

In the above equation, the effect of power was not included. معادلات با نادیده گرفتن موتور (پیشران) را شامل نمی‌شود.

It is for power-off flight only → این برای پرواز موتور خاموش است

$$\frac{dc_m}{dc_L} = 0$$ At low Mach numbers

Because for low M numbers & at the linear portion of $C_m - \alpha - C_L$ قسمت

$$\frac{dc_m}{dc_L} = \frac{\partial c_m}{\partial \alpha}\cdot\frac{\partial \alpha}{\partial c_L}$$

زیرا برای حالت M کم و قسمت خطی $C_m - \alpha - C_L$

In general

$$C_L = f(\alpha, M, T_c, ((1/2)\rho V^2)) \qquad C_m = f(\alpha, M, T_c, ((1/2)\rho V^2)$$

$$\frac{dC_m}{dC_L} = \frac{\partial C_m}{\partial \alpha}\frac{\partial \alpha}{\partial C_L} + \frac{\partial C_m}{\partial M}\frac{\partial M}{\partial C_L} + \frac{\partial C_m}{\partial T_c}\frac{\partial T_c}{\partial C_L} + \frac{\partial C_m}{\partial ((1/2)\rho V^2)}\frac{\partial ((1/2)\rho V^2)}{\partial C_L}$$

Pitching moment coefficient

$$M_{A_w} = (C_{m_0} + C_{m_\alpha}\alpha + C_{m_{i_H}} i_H + C_{m_{\delta e}}\delta e)\bar{q}s\bar{c}$$

Methods for evaluating $C_{m_\alpha}, C_{m_{i_H}}, \ldots$ are given in DATCOM.

62.I

Stability and control characteristics for steady state straight line flight

Control of an airplane can be achieved by providing an incremental lift force produced by one or more of the airplane's lifting surfaces.

Earliest method was to shift C.G by movement of the body.
Now use aerodynamic control surfaces.

62.II

Three primary aerodynamic control surfaces → سه سطح کنترل اولیه

-Pitch control by changing lift on either fwd. or aft. control surface, elevator

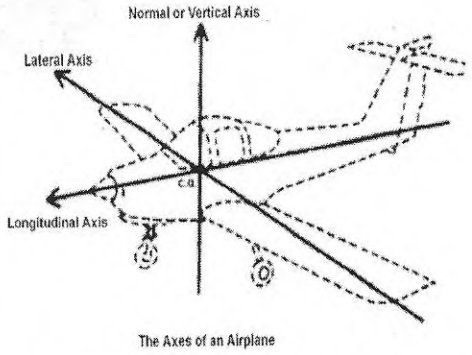
کنترل Pitch به وسیلهٔ تغییر لیفت در جلو و عقب - سطوح کنترل elevator

-Yaw control by deflecting rudder. → کنترل Yaw به وسیلهٔ منحرف کردن rudder

-Roll control by deflecting aileron in differential manner or deflecting one spoiler.

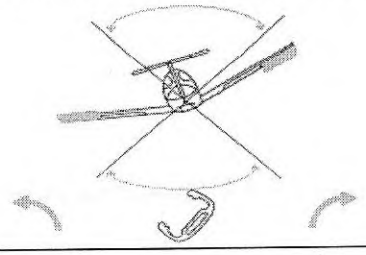
کنترل roll با منحرف کردن aileron در روشهای متفاوت و یا منحرف شدن یک spoiler

62.III

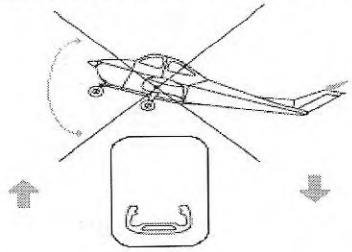


The Axes of an Airplane

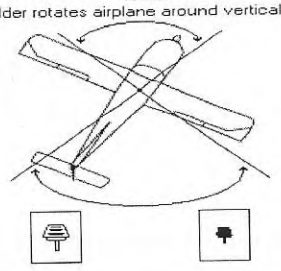
Directional Control - AILERONS
ROLL
Ailerons rotate airplane around longitudinal axis



Directional Control - Elevators
PITCH
Elevators rotate airplane around lateral axis



Directional Control - RUDDER
YAW
Rudder rotates airplane around vertical axis.



Factors affecting the design of a control surface are

-Control effectiveness:

Which is a measure of how effective the control deflection is in producing the desired control moment. It is a function of the size of the flap and tail volume ratio.

-Hinge moments:

Aerodynamic moment that must overcome to rotate the control surface. It governs the magnitude of force requirement of the pilot to move the control surface. Care must be taken so the control Force be within acceptable limit for the pilot.

-Aerodynamic and mass balancing:

Deal with techniques to vary the hinge moment so that the control stick forces stay within an acceptable range.

63.I

عوامل موثر طراحی یک سطح کنترل

اثرات کنترل :

یک مقیاس برای اینکه چطور انحراف کنترل در تولید کردن همان کنترل مورد نیاز موثر می‌باشد. این یک تابعی از اندازه نسبت حجم دم و فلپ است

همان آیرودینامیکی که باید بر چرخیدن سطوح کنترل غلبه کند. بزرگی نیرویی که مورد نیاز است که خلبان برای حرکت سطوح کنترل وارد کند را معین می‌کند. مراقب باید بود که نیروی کنترل داخل محدوده قابل قبول برای خلبان باشد

با تکنیکهایی سروکار داریم که hinge moment را تغییر می‌دهد که نیروی کنترل stick داخل یک محدوده قابل قبول باقی می‌ماند

IN THE NAME OF GOD

Flight Dynamics

Quiz II

1. List the Euler angle in order and give the axis which each one is rotated about.
2. Why is not Euler angles equal to yaw, pitch or roll angles?
3. Define S.S. flight and which terms is zero.
4. Reduce the following equation for a steady state symmetric pull-up.

$$X : m\,(\dot{U} - V_1 r_1 + q_1 W_1) = -mg\sin\theta_1 + \vec{F}_{Ax1} + \vec{F}_{Tx1}$$

$$Y : m\,(\dot{V} + U_1 r_1 - W_1 p_1) = mg\sin\Phi_1\cos\theta_1 + \vec{F}_{Ay1} + \vec{F}_{Ty1}$$

$$Z : m\,(\dot{W} - U_1 q_1 + V_1 p_1) = mg\cos\Phi_1\cos\theta_1 + \vec{F}_{Az1} + \vec{F}_{Tz1}$$

$$p_1 = \dot{\phi}_1 - \dot{\Psi}_1\sin\theta_1$$

$$q_1 = \dot{\theta}_1\cos\phi_1 + \dot{\Psi}_1\cos\theta_1\sin\phi_1$$

برای آنالیز نیروی کنترل مورد نیاز از معادلات اصلی پرواز خط مستقیم مورد استفاده قرار می‌گیرد.

To analyze the control power requirements, use the basic S.S. straight line flight equations derived earlier.

For S.S. rectilinear flight

Force equation :

X- Axis $$mg\sin\theta_1 = F_{A_{X_1}} + F_{T_{X_1}}$$

$$mg\sin\gamma_1 = -(c_{D_0} + c_{D_\alpha}\alpha_1 + c_{D_{i_h}} i_{h_1} + c_{D_{\delta e}}\delta_{e_1})\bar{q}_1 S + T_1\cos(\Phi_T + \alpha_1)$$

Y-Axis $$mg\sin\Phi_1\cos\theta_1 = -F_{A_{y_1}} + F_{T_{Y_1}}$$

$$-mg\sin\Phi_1\cos\gamma_1 = (c_{y_\beta}\beta_1 + c_{y_{\delta_A}}\delta_{A1} + c_{y_{\delta_R}}\delta_{R1})\bar{q}_1 S + Y_{T_1}$$

Z-Axis

$$mg\cos\Phi_1\cos\gamma_1 = (c_{L_0} + c_{L_\alpha}\alpha_1 + c_{L_{i_H}} i_{H_1} + c_{L_{\delta e}}\delta_{e_1})\bar{q}S + T_1\sin(\Phi_T + \alpha_1)$$

64.I

Moment equation :

$$0 = (C_{l_\beta}\beta_1 + C_{l_{\delta_A}}\delta_{A_1} + C_{l_{\delta_R}}\delta_{R_1})\bar{q}_1 Sb + l_{T_1}$$

$$0 = (C_{m_0} + C_{m_\alpha}\alpha_1 + C_{m_{i_H}} i_{H_1} + C_{m_{\delta_e}}\delta_{e_1})\bar{q}_1 S\bar{c} - T_1 d_T$$

$$0 = (C_{n_\beta}\beta_1 + C_{n_{\delta_A}}\delta_{A_1} + C_{n_{\delta_R}}\delta_{R_1})\bar{q}_1 Sb + N_{T_1}$$

All equations are in stability axis system $\therefore \quad \theta_1 = \gamma_1$

Assume q is known, also h is known too.

Unknown: $\gamma_1, \alpha_1, i_{H_1}, \delta_{e_1}, \Phi_1, \delta_{R_1}, \beta_1, \delta_{A_1}, T_1$ 9 of them

$\rho = const$

64.II

6 equations, 9 unknown

For most conditions, assume Φ , T, i_H are known

Now can solve them for other 6 unknowns.

- Split equations into longitudinal & lateral-directional groups.

For longitudinal case we have X, Z, axis & P.M.

Assume $\Phi_1 = 0$ (to get rid of Φ_1 in Y axis) &

uncouple the equations. Pre-select: $\rho, \Phi_1, T_1, i_{H_1}$

65.I

Longitudinal equations:

Assume: - un-powered glide

- Shallow glide $\Rightarrow \gamma$ is small; $COS\gamma \cong 1, Sin\gamma \cong \gamma$

$$mg\gamma = -(C_{D_0} + C_{D_\alpha}\alpha + C_{D_{i_H}} i_H + C_{D_{\delta_e}} \delta_e)\bar{q}S \quad \Rightarrow \quad \gamma, \alpha, \delta_e$$

$$mg = -(C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_h}} i_H + C_{L_{\delta_e}} \delta_e)\bar{q}S \quad \Rightarrow \alpha \ \& \ \delta_e \ only$$

$$0 = (C_{m_0} + C_{m_\alpha}\alpha + C_{m_{i_H}} i_H + C_{m_{\delta_e}} \delta_e)\bar{q}S\bar{c} \quad \Rightarrow \text{Moment trim}$$

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_e}} \\ C_{m_\alpha} & C_{m_{\delta_e}} \end{bmatrix} \begin{bmatrix} \alpha \\ \delta_e \end{bmatrix} = \begin{bmatrix} \frac{mg}{\bar{q}s} - C_{L_0} - C_{L_{i_H}} i_H \\ -C_{m_0} - C_{m_{i_H}} i_H \end{bmatrix}$$

65.II

Trim: force & moments are in equilibrium

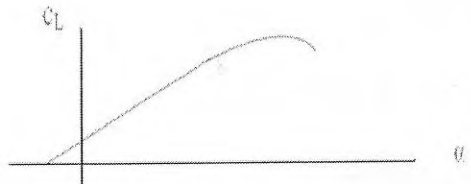
The trim value

$$\alpha = \alpha_{trim} = \frac{(C_{L_1} - C_{L_0} - C_{L_{iH}} i_H) C_{m_{\delta_e}} + (C_{m_0} + C_{m_{iH}} i_H) C_{L_{\delta_e}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}}$$

$$\delta_{e_{trim}} = \frac{-C_{L_\alpha} (C_{m_0} + C_{m_{iH}} i_H) - C_{m_\alpha} (C_{L_1} - C_{L_0} - C_{L_{iH}} i_H)}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}}$$

Knowing α & δ_e can solve for γ, flight path angle.

(66-I)



As long as α is within the linear range (no separation) and δ_e is deflected s.t. no flow is separated over H-T, also for shallow γ, with above equation the aircraft is said to be moment - trimmed in that S.S condition.

$$\frac{\partial \delta_e}{\partial C_L} = -\frac{C_{m_\alpha}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}} < 0$$

For good handling quality

(66-II)

Most book writes it as:

$$\frac{d\delta_e}{dC_L} = -\frac{\frac{dC_m}{dC_L}}{C_{m_{\delta_e}}}$$

when $C_{m_\alpha} = 0 \rightarrow \frac{\partial \delta_e}{\partial C_L} = 0 \rightarrow$ N.P

$C_{m_{\delta_e}}$: elevator power

(66-III)

A useful way of writing the above Soln. is:

$$\alpha_1 = \alpha_{C_{L_1}=0} + \frac{\partial \alpha}{\partial C_L} C_{L_1} \qquad \delta_{e_1} = \delta_{e_{C_{L_1}=0}} + \frac{\partial \delta_e}{\partial C_L} C_{L_1}$$

$\alpha_{C_{L_1}=0}$ is the A.O.A at which the aircraft is trimmed with . $C_{L_1} = 0$

$\delta_{e_{C_{L_1}=0}}$ is the elevator angle which the aircraft is trimmed with. $C_{L_1} = 0$

$\partial\alpha / \partial C_L$= the rate of change of trim angle of attack with C_L.

$\partial\delta_e / C_L$= the rate of change of elevator trim angle with C_L.

(67-A-I)

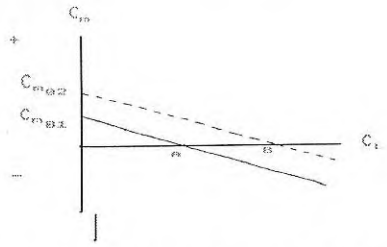
$$\alpha_{C_{L_1}=0} = \frac{(-C_{L_0} - C_{L_{i_H}} i_H) C_{m_{\delta_e}} + (C_{m_0} + C_{m_{i_H}} i_H) C_{L_{\delta_e}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}}$$

$$\delta_{e_{C_{L_1}=0}} = \frac{-C_{L_\alpha} (C_{m_0} + C_{m_{i_H}} i_H) - C_{m_\alpha} (-C_{L_0} - C_{L_{i_H}} i_H)}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}}$$

$$\frac{\partial \alpha}{\partial C_L} = \frac{C_{m_{\delta_e}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}}$$

(67-A-II)

Longitudinal control:



Aircraft is trimmed when $C_m = 0$ at a C_L of corresponding to point A if can trim only at one point it is no good.

To slow the aircraft down, and operate at a higher $C_L \Longrightarrow$ must provide additional $C_m = C_{m_{02}}$

(64-A-I)

هواپیما trim می‌شود وقتی که $C_m = 0$ در C_L مشابه نقطه A
اگر فقط بتوان در یک نقطه trim کرد این اصلاً خوب نیست
برای هواپیمایی که سرعت کم کرده و عمل می‌کند در یک C_L بالاتر
$C_m = C_{m_{02}}$ ← باید C_m اضافه فراهم آورد

$$C_L = \frac{2W}{\rho V^2 S}$$

هواپیما باید در حالتهای مختلف trim شود

To do that:

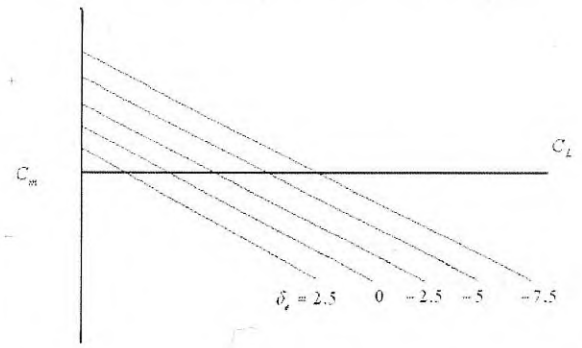
* Change the wing pitching moment. (change camber & wing twist) use leading or T.E flaps but these devices are for lift control ; tailless aircraft use L.E or T.E flaps.
* Change X_{ac}/c used by current "hang-glider" limitations on C.G shift and stability level.
* Change tail A.Θ.A by moving entire tail

" stabilator " or by moving a hinged surface at the T.E. "Elevator"

Stabilator is used when C.G has considerable fwd. travel or the wing pitching moment about the a.c. is large (supersonic flight)

(64-A-II)

Can have a series of C_m vs. C_L curves corresponding to each elevator angle $\Rightarrow$ get many " trim pts ".

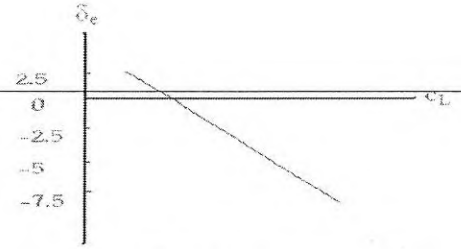


Can get the range of useful c_L at some c.g position

When $\delta_e > 0 \Rightarrow$ positive $C_{L_{\delta_e}}$ and negative $C_{m_{\delta_e}}$.
Elevator does not change the slope of the P.M curve.

(64-B-I)

The magnitude of the moment coeff. obtained per degree deflection of the elevator is termed the elevator power.

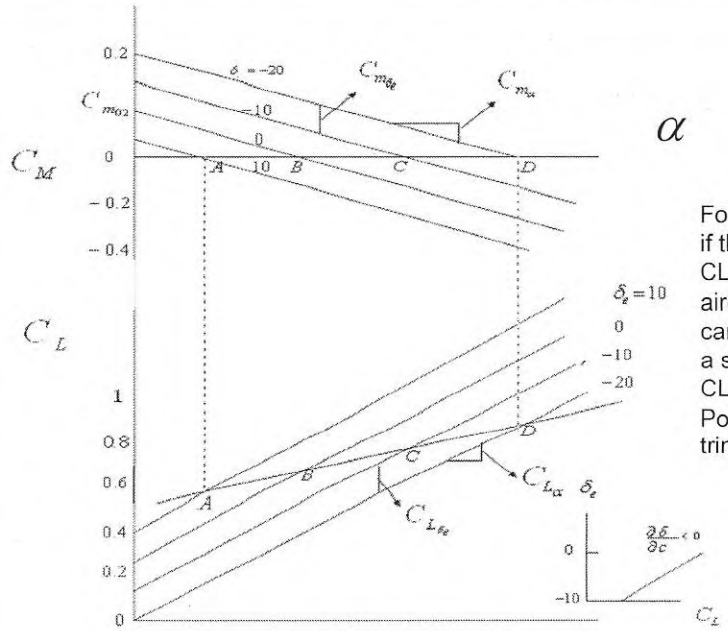


$$\frac{\partial \delta_e}{\partial C_L} < 0$$

Now we obtain this analytically:

$$\delta_e = \delta_{e_0} + \frac{d\delta_e}{dC_L} C_L$$

(64-B-II)



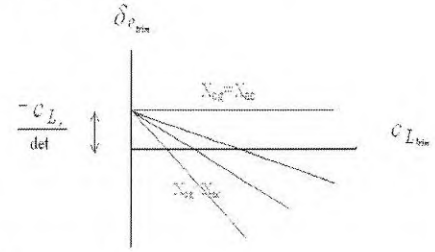
For point D, high CL, if this is the highest CL that can trim the aircraft, aircraft cannot be trimmed at a speed below this CL.
Point A, min trimmable CL

Trimmed C_L vs. α (67-B-I)

Two pts:

- Imposed elevator deflection range of +10 to -20 deg.
- Magnitude of control power which is α to the vertical distance between the constant elevator lies in the upper part of fig.

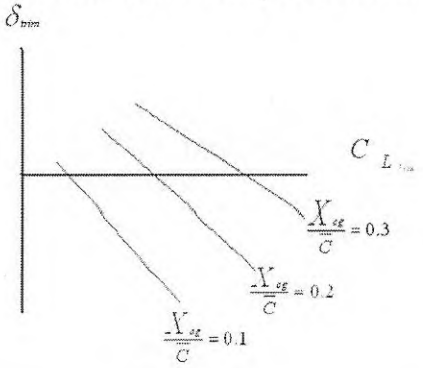
Both factor are under the control of the designer & must be selected in such a way that trim is not a limiting factor anywhere in the intended flight envelope of the aircraft.



(67-C)

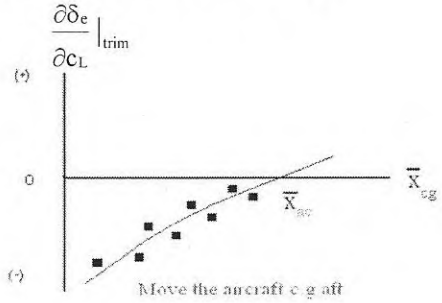
Flight test measurement of $\overline{X}_{ac}$ or $\overline{X}_{cg}$ to get location of aircraft a.c:

-At several C.G positions and various airspeed measure $\delta_e = \delta_e (C_L)$



(67-I)

- Determine $\partial\delta_e/\partial C_L\,|_{trim}$ for each C.G location and extend the curve until $\partial\delta_e/\partial C_L\,|_{trim}=0$ $\bar{X}_{ac}$ neutral point stick fixed.(N.P.fix)



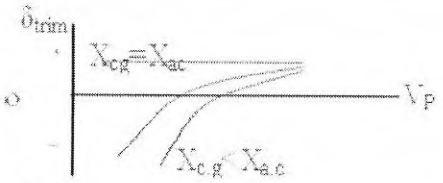
$$\frac{d\delta_{e_{trim}}}{dC_{L_{trim}}} = -\frac{C_{L_\alpha}}{\det}(\bar{X}_{cg} - \bar{X}_{ac})$$

(67-II)

Change in δ_e with speed:

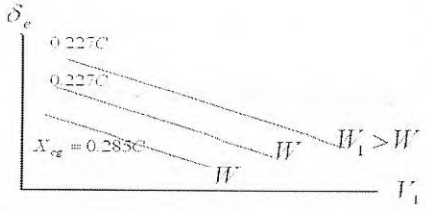
In the absence of compressibility, aerolastic effect, propulsive effects variation of δ_{trim} with V_p is simple.

(67-III)



For $i_H=0$ & α_1 is known

$$\delta_{trim} = -\frac{C_{m_0} C_{L_\alpha} + C_{m_\alpha} C_{L_{trim}}}{C_{m_{\delta_e}} C_{L_\alpha} - C_{m_\alpha} C_{L_{\delta_e}}}$$



Assume L=W $\Longrightarrow$

$$C_L = \frac{W}{qS} = \frac{W}{\frac{1}{2}\rho V_p^2 S}$$

(68-I)

$$\frac{\partial \delta_e}{\partial V_p}\Big|_M = \frac{\partial \delta_e}{\partial C_L}\Big|_M \frac{\partial C_L}{\partial V_p}$$

At low M

$$\frac{\partial C_L}{\partial V_P} = -\frac{4W}{\rho S V_P^3} \Rightarrow \frac{\partial \delta_e}{\partial V_P})_M = -\frac{C_{m_\alpha}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}} .(-\frac{4W}{\rho S V_P^3})$$

$$\left|C_{L_\alpha} C_{m_{\delta_e}}\right| >> \left|C_{m_\alpha} C_{L_{\delta_e}}\right| \qquad \because C_{m_{\delta_e}} < 0$$

$\Rightarrow \frac{\partial \delta_e}{\partial V_p}\Big|_M > 0$ as long as $C_{m_\alpha}<0$ is satisfied

As long as C.G. is ahead of a.c., but when C.G is behind a.c. $C_{m_\alpha} > 0$

(68-II)

∴ To ↑ airspeed from V_{P_1}, must move the elevator T.E. down.

To ↓ airspeed from V_{P_1}, must move the elevator T.E. up.

Mach number effect:

$$\frac{d\delta_{trim}}{dV_p} = \frac{\partial \delta_{trim}}{\partial C_L}\frac{\partial C_L}{\partial V_p} + \frac{\partial \delta_{trim}}{\partial M}\frac{\partial M}{\partial V} > 0$$

>0 As long as $C_{m_\alpha} < 0$

(68-III)

$$\delta_{e_{trim}} = -\frac{c_{m_0} + c_{m_\alpha}\alpha_{trime}}{c_{m_{\delta_e}}} \qquad i_h = 0$$

$$\frac{\partial \delta_e}{\partial V_p} = \frac{\partial M}{\partial V_p} \times \frac{\partial \delta_e}{\partial M} = \frac{1}{a}\frac{\partial \delta_e}{\partial M} =$$

$$-\frac{1}{a}\frac{\frac{\partial c_{m_0}}{\partial M} + \frac{\partial c_{m_\alpha}}{\partial M}\alpha_{trime} + c_{m_\alpha}\frac{\partial \alpha_{trime}}{\partial M}}{c_{m_{\delta_e}}} + \frac{1}{a}\frac{c_{m_0} + c_{m_\alpha}\alpha_{trime}}{c_{m_{\delta_e}}^2}\frac{\partial c_{m_{\delta_e}}}{\partial M}$$

69-I

- $\frac{\partial \delta_e}{\partial V_p}$ depends on

$$\frac{\partial c_{m_0}}{\partial M}, \frac{\partial c_{m_\alpha}}{\partial M}, \frac{\partial \alpha_{trim}}{\partial M}, \frac{\partial c_{m_{\delta e}}}{\partial M}$$

- Neglecting the second term:

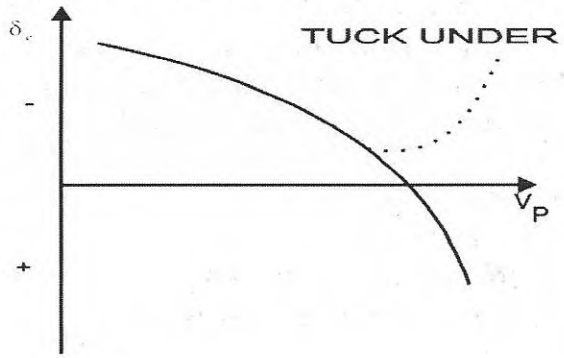
$$c_{m_u} \approx \frac{1}{a}\left[\frac{\partial c_{m_0}}{\partial M} + \frac{\partial c_{m_\alpha}}{\partial M}\alpha_{trim}\right]$$

- tuck derivative

69-II

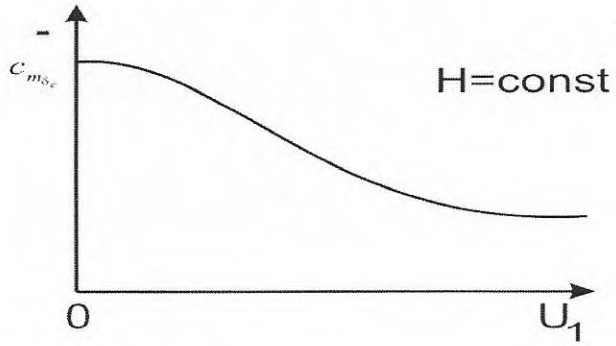
- If $c_{m_u} \rangle 0 \Rightarrow \frac{\partial \delta_e}{\partial V_p}$ is more stable
- For $c_{m_u} \langle 0$ it may change the sign of $\frac{\partial \delta_e}{\partial V_p}$ at high speed

- "***Tuck under***" happens due to Mach effect.
- Must design the aircraft to avoid critical Mach. Cause speed upset.



69-III

- ***AEROELASTIC EFFECT:***



- Aft – fuselage bending ↓ $C_{m_{\delta_e}}$ with u_1

69-V

IN THE NAME OF GOD

Flight Mechanics II

- ***UNPOWERED GLIDE:***
- USE c_L as the independent variable

$$c_L = c_{L_0} + c_{L_\alpha}\alpha + c_{L_{i_h}} i_h + c_{L_{\delta_e}}\delta_e \qquad (1)$$

$$c_m = c_{m_0} + c_{m_\alpha}\alpha + c_{m_{i_h}} i_h + c_{m_{\delta_e}}\delta_e$$

$$c_{m_\alpha} = c_{L_\alpha}(\bar{X}_{CG} - \bar{X}_{a.c}) \Rightarrow \frac{c_{m_\alpha}}{c_{L_\alpha}} = \bar{X}_{CG} - \bar{X}_{AC} = -S.M$$

$$C_{m_{\delta E}} = \frac{\partial c_m}{\partial \delta_e})_{i_h \,\&\, \alpha = const}$$

,

$$\overline{c}_{m_{\delta_e}} = \frac{\partial c_m}{\partial \delta_e})_{c_L = const \,,\, i_H = const}$$

- Solve for

$$\alpha \Rightarrow$$

$$\alpha = \frac{c_L - c_{L_0} - c_{L_{i_h}} i_h - c_{L_{\delta_e}} \delta_e}{c_{L_\alpha}}$$

70-II

$$c_m = c_{m_0} + c_{m_\alpha} \frac{c_L - c_{L_0} - c_{L_{i_h}} i_h - c_{L_{\delta_e}} \delta_e}{c_{L_\alpha}} + c_{m_{i_h}} i_h + c_{m_{\delta_e}} \delta_e$$

$$c_m = \overline{c}_{m_0} + \frac{\partial c_m}{\partial c_L} c_L + \overline{c}_{m_{i_h}} i_h + \overline{c}_{m_{\delta_e}} \delta_e \qquad (2)$$

- (1), (2) $\Rightarrow$ $\overline{c}_{m_0} = c_{m_0} - \frac{c_{m_\alpha}}{c_{L_\alpha}} c_{L_0}$, $\overline{c}_{m_{\delta_e}} = c_{m_{\delta_e}} - \frac{c_{m_\alpha}}{c_{L_\alpha}} c_{L_{\delta_e}}$

$$\overline{c}_{m_{i_h}} = c_{m_{i_h}} - \frac{c_{m_\alpha}}{c_{L_\alpha}} c_{L_{i_h}}$$

70-III

$$\bar{c}_{m_{\delta_e}} = \left(\frac{\partial c_m}{\partial \delta_e}\right)_{i_h = const \;\; c_L = const}$$

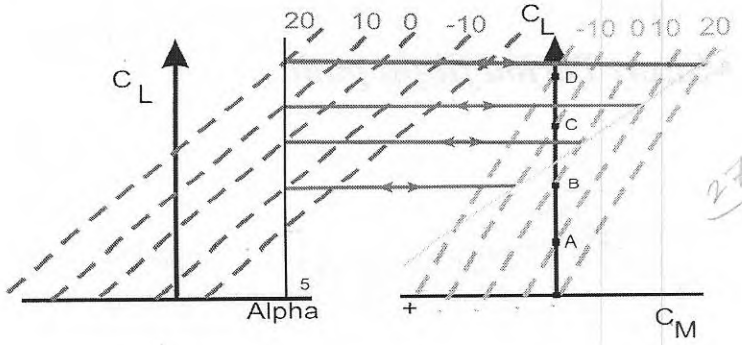
$$\bar{c}_{m_{i_h}} = \left(\frac{\partial c_m}{\partial i_h}\right)_{\delta_e = const \;\; c_L = const}$$

$$\bar{c}_{m_0} = c_{m(c_L = i_h = \delta_e = 0)}$$

70-IV

$c_m - c_l - \alpha$ *Representation:*

- solve for equation (1), (2) at $\bar{x}_{c.g} = .25\ c$
- For $\bar{x}_{c.g} = .25\ c, \delta_e = 0, \alpha = 5$ we will have:



- A, B, C, D are trim points for different δ_e

71-I

- $c_{l_{trime}}$ with C.G and $\bar{x}_{c.g.} = .25$ along $c_{m_{25\bar{c}}} = 0$

- Good to analyze wind tunnel data and study the effect of c.g changes on S.C. chord.

- line for constant A.O.A of $\alpha = 5$, can calculate $\left.\frac{\partial c_L}{\partial \delta_e}\right|_{\alpha=5}$

71-II

- also can get $\left.\frac{\partial c_m}{\partial \delta_e}\right|_{c_L = con}$

- can determine $c_{m_{\delta_e}}$ from the constant α lines:

$$c_{m_{\delta_e}} = \frac{\partial c_m}{\partial \delta_e})_{i_h=const,\alpha=con}$$

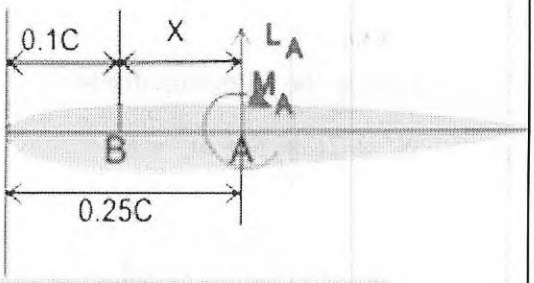
71-III

- Also can calculate c_L for trim values for i

orbit C.G lo

$$c_{m_{.25\bar{c}}} = \bar{c}_{m_0} \Big|_{.25\bar{c}} + \frac{dc_m}{dc_L} \Big|_{.25\bar{c}} c_L + \bar{c}_{m_{\delta_e}} \Big|_{.25\bar{c}} \delta_e, \qquad (1)$$

$$i_h = 0 \qquad L_b = L_a$$



$$M_B = M_A - L_A X$$

$$M_{0.1\bar{c}} = M_{0.25\bar{c}} - L_{0.25\bar{c}}(0.25 - 0.1)\bar{c}$$

71-IV

$$c_{m_{0.1\bar{c}}} = \bar{c}_{m_0} \Big|_{0.25\bar{c}} + \frac{dc_m}{dc_L})_{.25\bar{c}} c_L + \bar{c}_{m_{\delta_e}} \Big|_{0.25\bar{c}} \delta_e - (0.15)c_L$$

$$c_{m_{0.1\bar{c}}} = \bar{c}_{m_0} \Big|_{0.25\bar{c}} + [\frac{dc_m}{dc_L} \Big|_{0.25\bar{c}} - 0.15]c_L + \bar{c}_{m_{\delta_e}} \Big|_{0.25\bar{c}} \delta_e$$

Can see that:

$$\frac{dc_m}{dc_L} \Big|_{0.1\bar{c}} = \frac{dc_m}{dc_L} \Big|_{0.25\bar{c}} - 0.15$$

72-I

- If want to trim at $\bar{x}_{cg} = 0.1\bar{c} \Rightarrow c_{m_{0.1\bar{c}}} = 0$

$$\Rightarrow 0 = \bar{c}_{m_0})_{0.25c} + [\frac{dc_m}{dc_l})_{0.25c} - 0.15]c_l + \bar{c}_{m_{\delta_e}})_{0.25c}\delta_e \qquad (2)$$

72-II

- Subtract (1) from (2): $c_{m_{0.25\bar{c}}} = 0.15c_L$

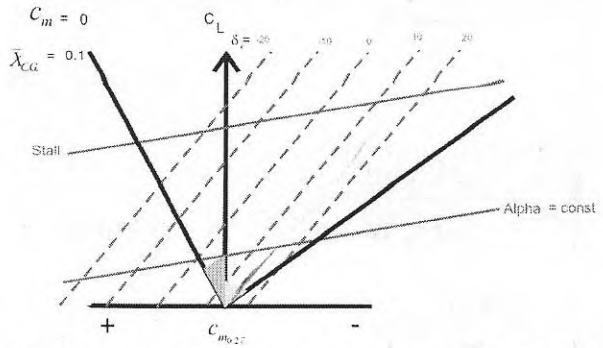
- ***Example***:

- To trim at $\bar{x}_{cg} = 0.3 \Rightarrow c_{m_{0.25c}} = -0.05\,c_l$

- To trim at $\bar{x}_{cg} = 0.15 \Rightarrow c_{m_{0.25c}} = 0.1c_l$

72-III

- Can read complete trim condition within the any C.G. range for any Aircraft:



- Straight line: trim line about 0.1 chord whit C.G. forward

72-IV

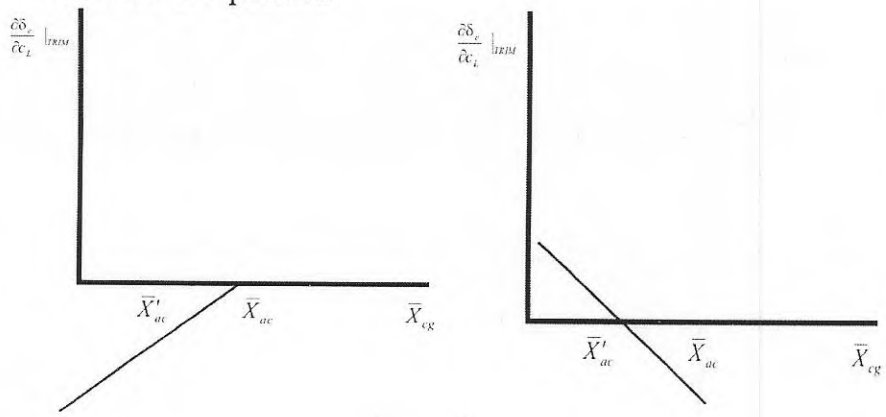
- *The forward limit:*

- As c.g moves forward the stability of aircraft increases
  $\Rightarrow$ larger control movement and force are required to maneuver or change the trim condition .
- The forward cg limit is based on control considerations and may be determined by:
  1- The control force /g shall not exceed a specified value.
  2- The control force gradient at trim . $\frac{\partial p}{\partial V}$. shall not. exceed a specified value.
  3- The control force requirement to land from trim at the approach speed, shall not exceed the specified value.
  4- The elevator angle required to land shall not exceed max up elevator.
  5- The elevator angle required to raise the nose wheel off the ground at take off shall not exceed the max up elevator

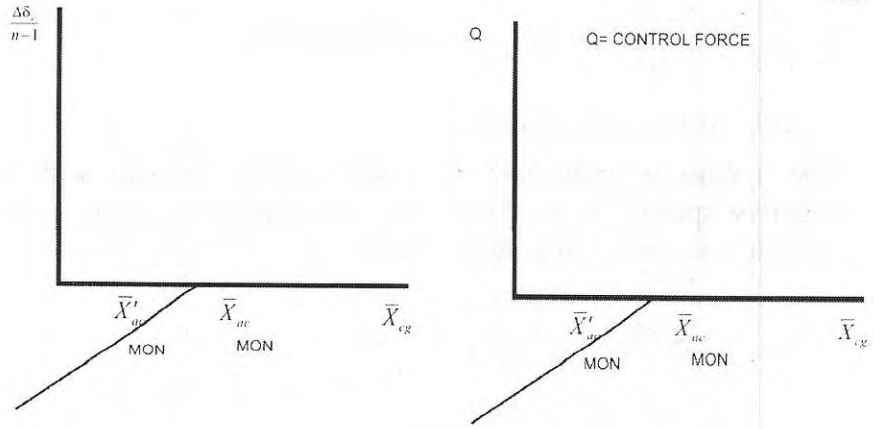
72-A-I

- *Most after C.G.*

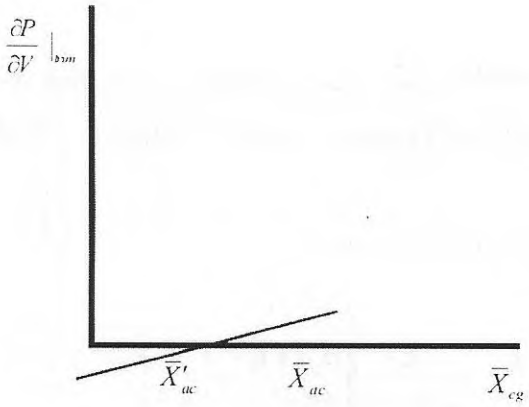
- The most after C.G. location is determined by longitud. stability and control sensitivity. Can use SAS if want to have rearward C.G.position.



- 72-A-II



72-B-I



72-B-II

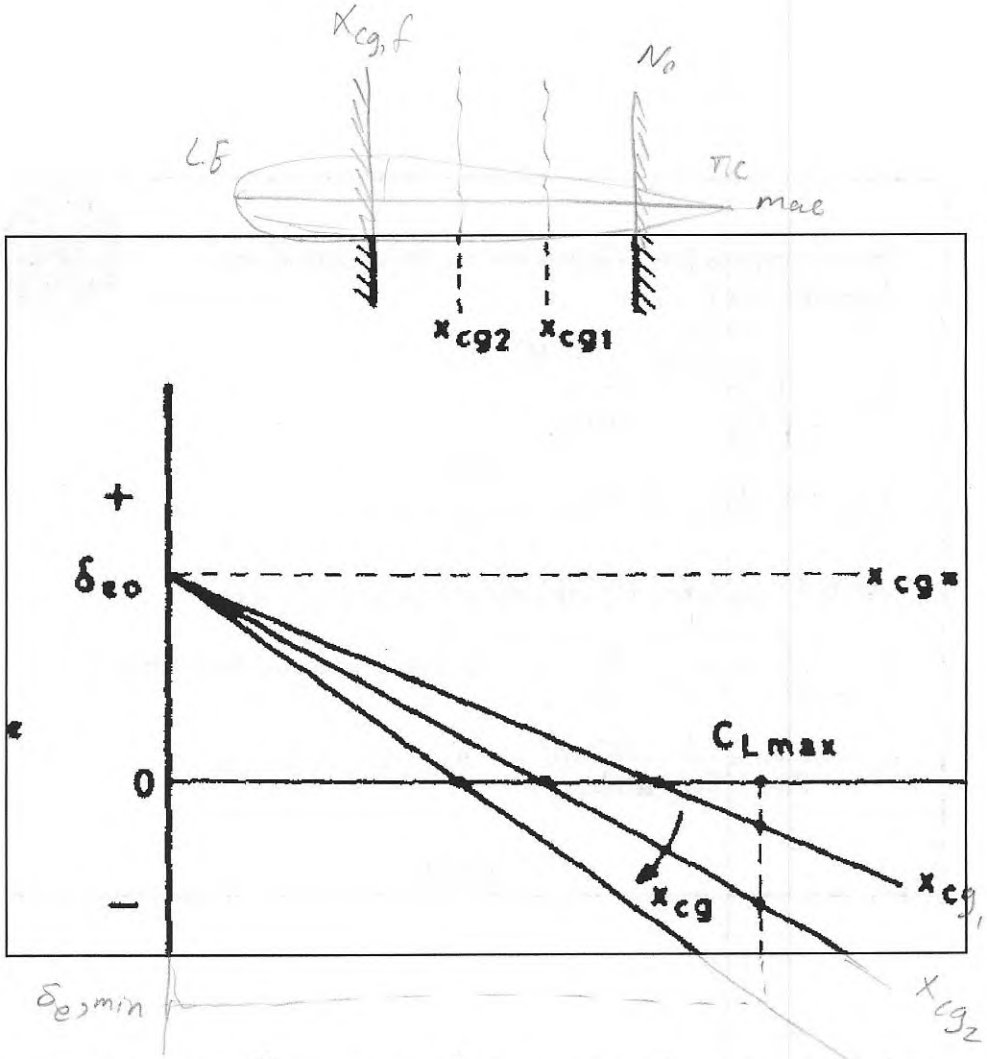
- None of the above gradient should be reversed, have
- signs associated with low value of $\bar{x}_{c.g}$

- If the controls are reversed $\Rightarrow \bar{x}_{c.g} \langle \bar{x}_{a.c}$

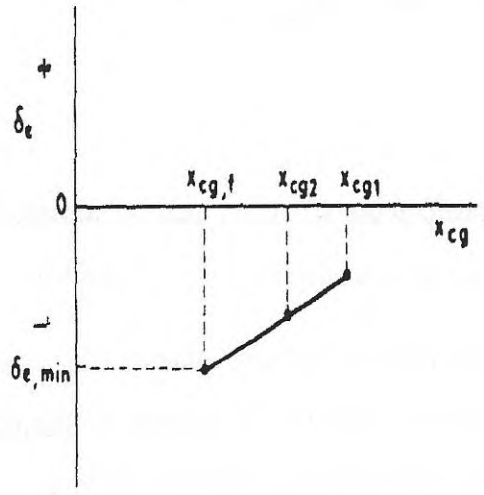
- The neutral point gives the stability boundary
- IN general:

- Never load the aircraft such that it s C.G. lies aft of N.P. It is where $\frac{dc_m}{dc_L} = 0$ or has its min value.

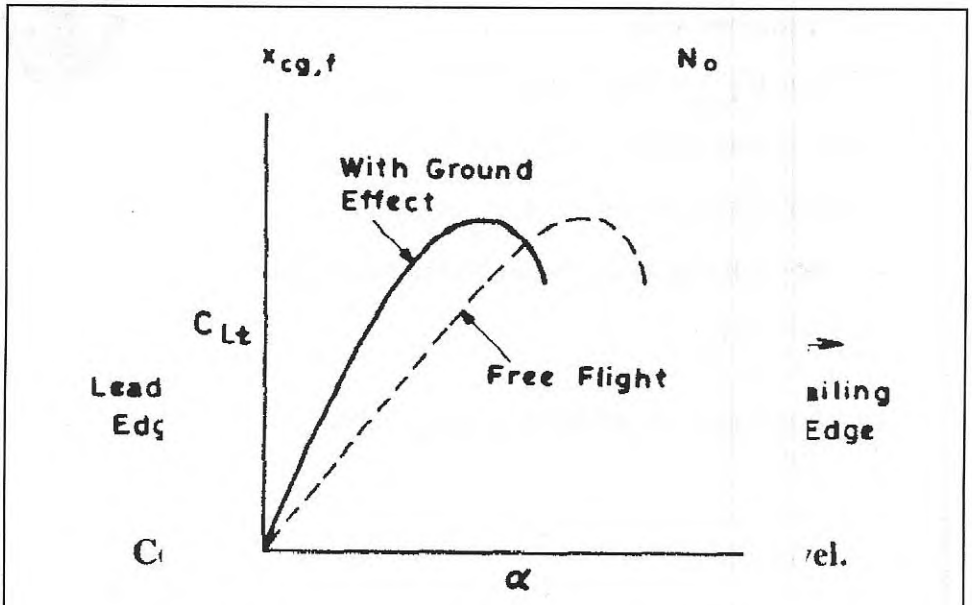
72-B-III



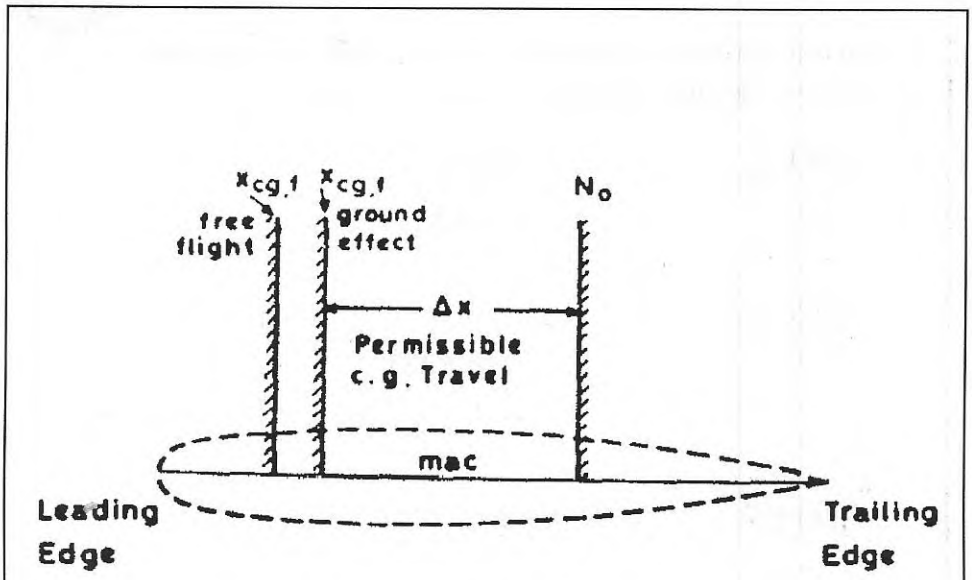
Effect of center of gravity location on elevator deflection for trim.



Elevator required for trim at $C_{L,max}$.

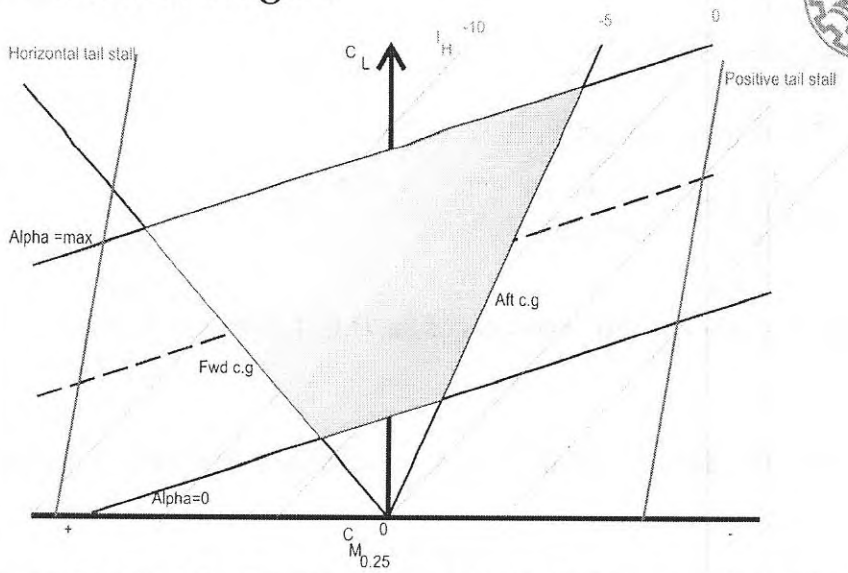


Schematic illustration of ground effect.



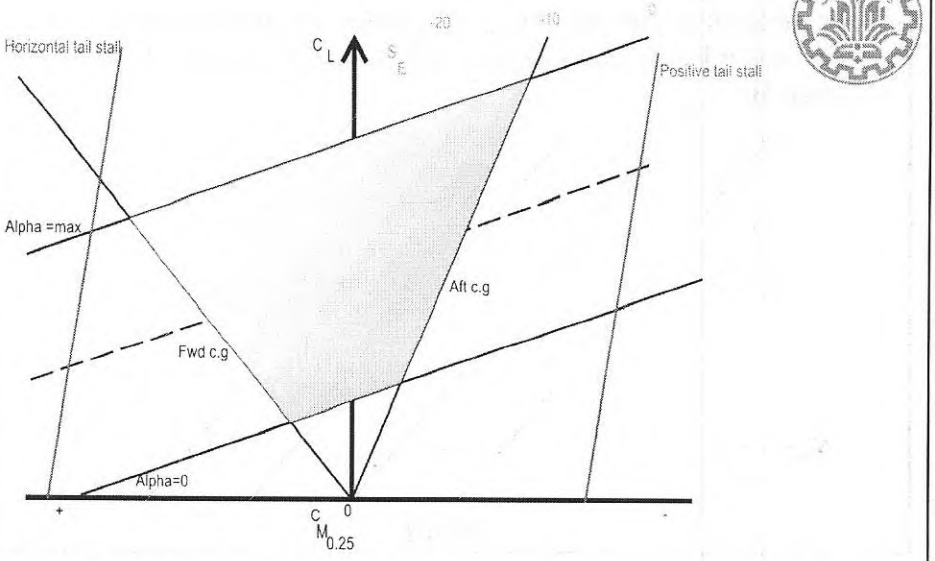
Permissible center of gravity travel with ground effect.

- ***Trim triangle:***



- ***Variable incidence stabilizer***

72-C-I



Variable elevator angle

72-C-II

trim triangle:

- The triangle area bounded by the forward and aft C.G. lines and by the max A.O.A.
- An A/C must be trimmable in this triangle with reasonable stabilizer incidence value (the reasonable is whether or not tail is stalled)
- the triangle is
- Locus of zero P.M. coefficients at fwd C.G.
- Locus of zero p.m coefficients at aft C.G
- Max. allowable aircraft a.o.a (either by the aircraft stalls a.o.a or by that a.o.a location along which the aircraft P.M. becomes unmanageable. Tail stall should never occur within the triangle trim.

72-C-III

• *Down wash:*

- How much can move C.G forward? →

or

- What is the locus of line in the trim diagram along which H.T is stalled?

- $\alpha_{h_{stall}}$ is obtained by :

$$\alpha_h = \alpha - \varepsilon + i_h = \alpha_{h_{stall}} = \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + i_h - \varepsilon_0$$

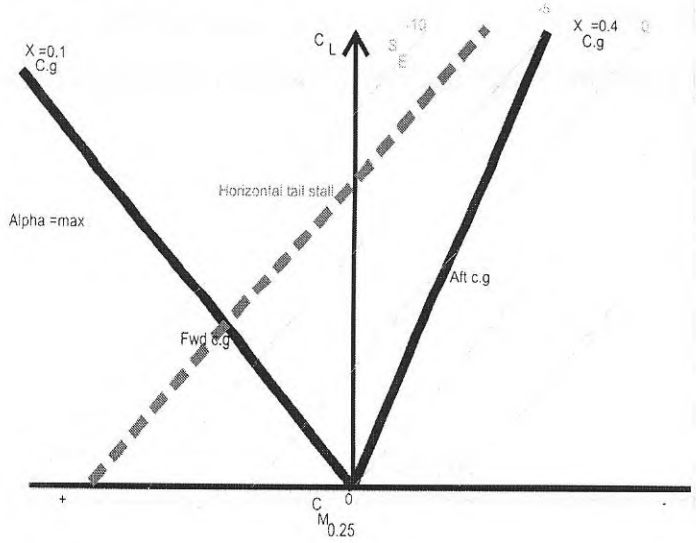
73- I

- $\alpha_{tail_{stall}} = \dfrac{\alpha_h - i_h + \varepsilon_0}{1 - \dfrac{d\varepsilon}{d\alpha}}$ aircraft A.O.A for which the tail is always stall.

زاویهٔ حملهٔ هواپیما که دم همیشه استال می‌کند

73-II

- - For a given tail stall a.o.a, $\alpha_{h_{stall}}$ can constract α_0 Versus i_h location.



- ***If tail – stall becomes a problem increasing tail effectiveness:***

1. Negative camber: increases - magnitude of $\alpha_{h_{stall}}$ while decreases + magnitude of. $\alpha_{h_{stall}}$

- Many current jet aircraft employ negative cambered tail.

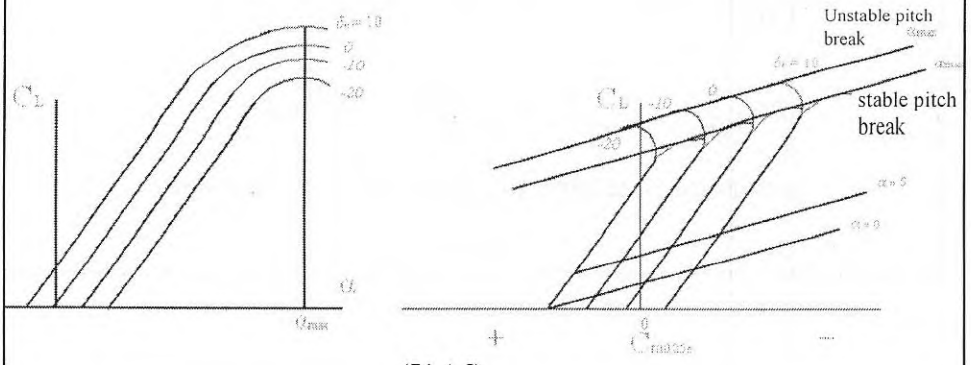
2. Fix L.E slots: increasing - magnitude of $\alpha_{h_{stall}}$ while decreases + magnitude of $\alpha_{h_{stall}}$.(Cessna cardinal)

3. Lower side leading edge blowing: : increasing - magnitude of $\alpha_{h_{stall}}$ while decreasing H.T + magnitude of $\alpha_{h_{stall}}$ (F-4)

73-IV

- The trim diagrams are all for low α case (linear variation)
- All trim diagram apply to tail-aft configuration.

Stable & Unstable pitch breaks



(74-A-I)

•Flow separation occur first on the top surface of inboard wing

•If at α_{stall} pitching moment line break to the right , A/C nose down , called "Stable Pitch Break"

If at α_{stall} pitching moment line break to the left , A/C nose up , called "Unstable Pitch Break"

α_{max} for Unstable Pitch Break $<$ α_{max} for stable Pitch Break
the difference in α_{max} depends on pitch controllability in that region.

Also lower αmax imply higher reference speed & thus longer field-lengths

(74-A-II)

How to get ε from tunnel test ?

$$\alpha_H = \alpha - \varepsilon + i_H \quad ; \quad \alpha_H = 0 \Rightarrow \varepsilon = \alpha + i_H \Rightarrow C_{L_{wB}} = C_{L_{wBH}}$$

For a symmetric tail

Adjust i_H until ;

$$C_L = C_{L_{wB}} \Rightarrow \alpha_H = 0 : \varepsilon = \alpha + i_H$$

(74-I)

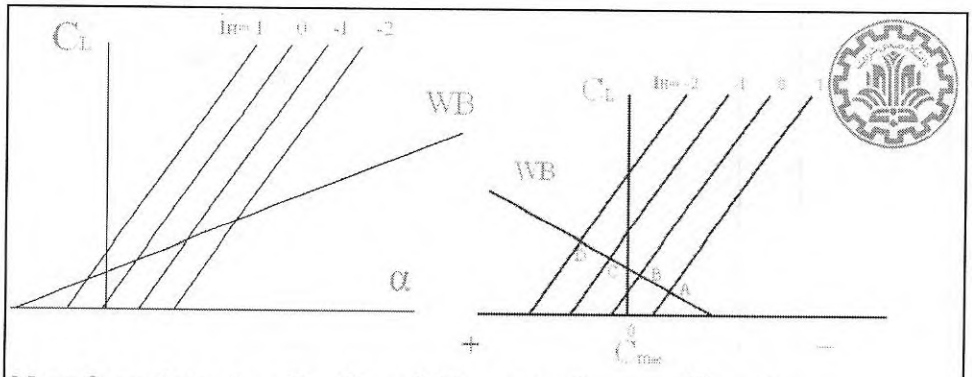
Tunnel test

1-Install entire A/C ; determine C_L - C_m - α

2-Remove tail $\Rightarrow C_{m_{wB}}$; $C_{L_{wB}}$ *vs* α

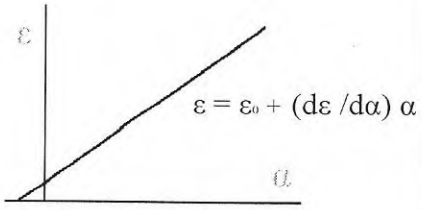
3-Use data find $\begin{cases} C_L = C_{L_{WB}} \\ \alpha_H = 0 \Rightarrow \varepsilon = \alpha + i_H \end{cases}$ $\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha$

(74-II)



Now for points A , B , C and D $\alpha_H = 0 \longrightarrow C_{LH} = 0$

At each point read α , $\varepsilon = \alpha + i_H$

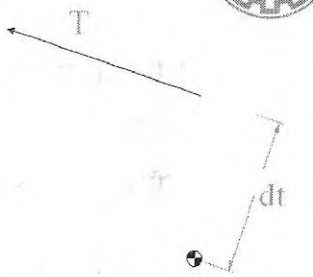


Example at A

$\alpha = -1$ $i_H = 1 \longrightarrow$ $\varepsilon = 0$

(74-III)

Thrust Effect



$$0 = \left(C_{m_\alpha}\alpha + C_{m_{iH}} i_H + C_{m_{\delta e}}\delta_e + C_{m_0}\right)\bar{q}s\,\bar{c} - TdT$$

If T = const $\Longrightarrow$

$$0 = \left(C_{m_0} - \frac{TdT}{\bar{q}s\bar{c}} + C_{m_\alpha}\alpha + C_{m_{iH}} i_H + C_{m_{\delta e}}\delta_e\right)\bar{q}s\,\bar{c}$$

terms like $C_{m\alpha}$, $C_{m\delta e}$ and C_{miH} = f(T)

(75-I)

Lateral static stability

S.S. rectilinear flight

y_s force (side force)

$$-mg\sin\varphi\cos\gamma = \left(C_{y_\beta}\beta + C_{y_{\delta A}}\delta_A + C_{y_{\delta R}}\delta_R\right)\bar{q}s + F_{y_T}$$

Roll
$$0 = \left(C_{l_\beta}\beta + C_{l_{\delta A}}\delta_A + C_{l_{\delta R}}\delta_R\right)\bar{q}s\,b + L_T$$

Yaw
$$0 = \left(C_{n_\beta}\beta + C_{n_{\delta A}}\delta_A + C_{n_{\delta R}}\delta_R\right)\bar{q}s\,b + N_T$$

Variables φ, γ, β, δ_A, δ_R, T
γ : known from longitudinal
T : fixed
$\therefore$ Other unknowns φ, β, δ_A and δ_R

(75-II)

Directional stability and control

-Talked about stability of A/C when flying in equilibrium on symmetrical flight paths.
Control over A. O. A. ⟶ lift coefficient & insuring static stability at this A. O. A.

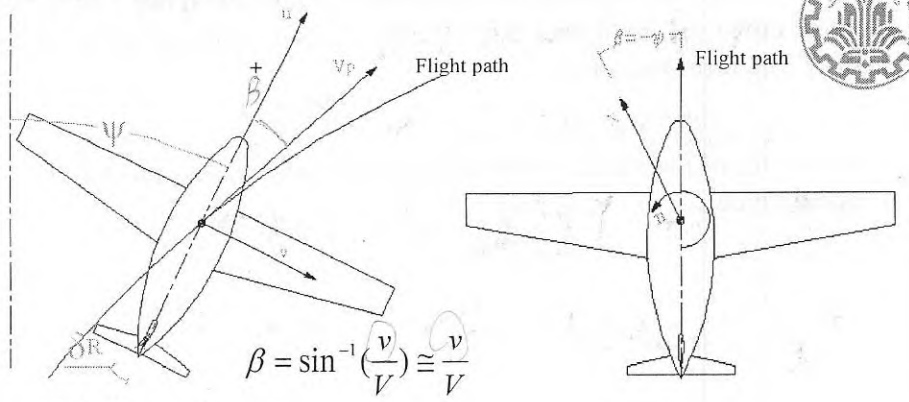
-If the flight path is not symmetric ⟶ relative wind makes some angle to the A/C's centerline ⟶ β (sideslip angle)

β is like α except ; α determines C_L & very closely V_∞.

Sideslip angle, β , is used to ↑ drag, and the flight path angle during its landing and can be used for smooth acrobatic such as slow roll, used when having asymmetric power.

(76-I)

Desire to have β=0



β - Sideslip angle
Ψ - Angle of yaw

For curved flight path, ψ ,the angular displacement of the A/C's centerline from some azimuth direction, taken as zero at some instant of time (ψ≠β). For a 360 turn, A/C has yawed 360, with β=0

(76-II)

The A/C is defined to have a positive sideslip when the y -component of A/C velocity relative to the surrounding air is positive.

∴ a positive β , a left hand rotation of A/C about the yaw axis is required.

Note φ and ψ are different from α & β

α & β are called aerodynamic angles since they are defined w.r.t. A/C's velocity vector relative to the surrounding air.

Φ & ψ are orientation angles i.e. kinematic angles, define component of A/C orientation relative to the earth & are independent of V_∞.

(76-A-I)

Ψ is measurement of the change in heading or orientation of the A/C relative to the earth. It is the angle between the A/C's plane of symmetry and a reference plane fixed in space, Fig a.

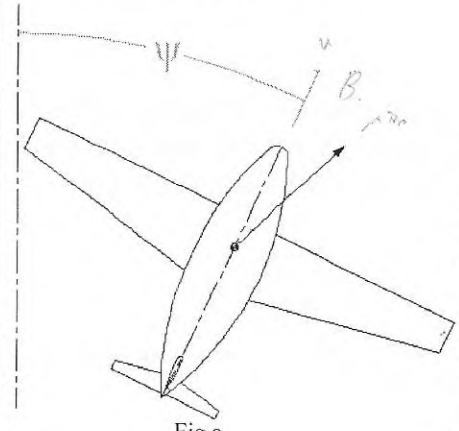


Fig a

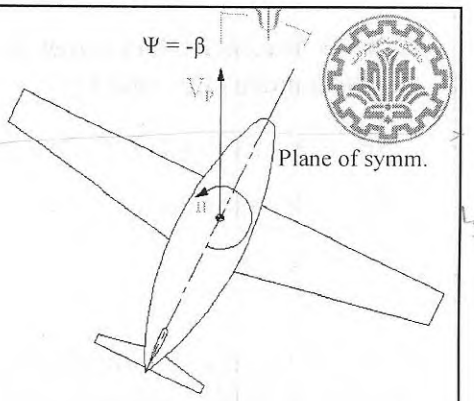


Fig b

If the direction of motion remains unchanged, but the A/C is yawed then β & ψ are related by ψ = -β ,Fig b.

Note aerodynamic forces &moments do not depend on ψ

(76-A-II)

. Analysis of the longitud. Motion is relatively simple since it deals with rotational about one axis (y axis) ➡ the rotation stiffness about y axis is a very imp. Criterion for the dynamic behavior.

. For the lateral motion rotation takes place about 2 axis : X and Z The moments assoc. with these rotations are cross-coupled, i.e roll rotation ,p, produces a yawing moment C_n as well as the rolling moment C_l

. Yaw displacement β and rate r both produce rolling and Yawing moment also roll and Yaw controls are often cross-coupled .

76-B-I

. Further deflection of aileron δA can produce significant Yawing moment .

. Deflection of rudder δr can produce significant rolling moment .

76-B-II

Another imp. Diff. is that in normal flight all lateral motion and force variables are zero ➡ no fundamental trimming problem i.e aileron and rudder are nominally undeflected . In actuality these controls or inertial asymmetry exists when :

. one engine is off .
. multi propellers all rotating the same way .
. etc.

. The C.G. position is not a dominant parameters for lateral charact . since the gravity vector in normal flight lies in the plane of symmetry ➡ C.G. limits are governed by considerations of derivatives from the longitud. charact .

76-B-III

Side slip angle β gives rise to :

- Rolling moment ℓ , ℓ_A

-Yawning moment $N' = N_A$

- Side force $F_{Ay} = F_{Ay}$

All of the above depend on M , Re , α ,β , P, control deflection of Aileron, Spoiler, Rudder, etc .

(77-I)

Rolling moment :

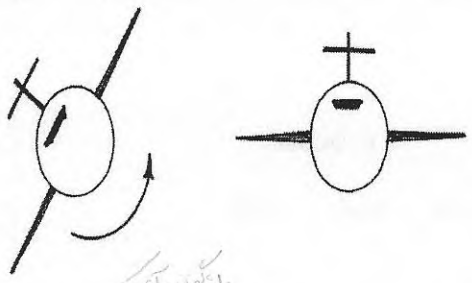
$\ell_A = C_\ell \; q \; s \; b$

C_ℓ : total A/C rolling moment coeff.

An A/C possesses static roll stability if , when it is disturbed from a wing level altitude a restoring moment is developed.

Major control over the rolling moment is aileron system .

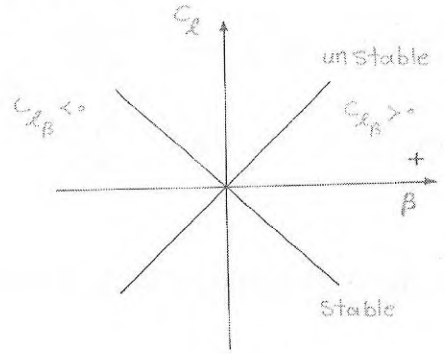
(77-II)



" Roll upset "

(77 – III)

An A/C has a stable dihedral effect C_{ℓ_β} if a negative rolling moment (left wing down) is created as a result of positive side slip β.



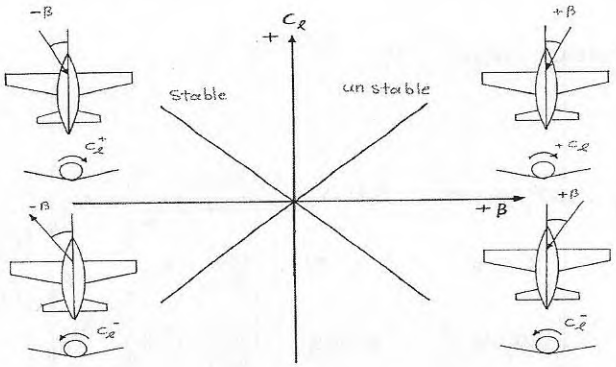
(77 – IV)

The roll moment depend up on :

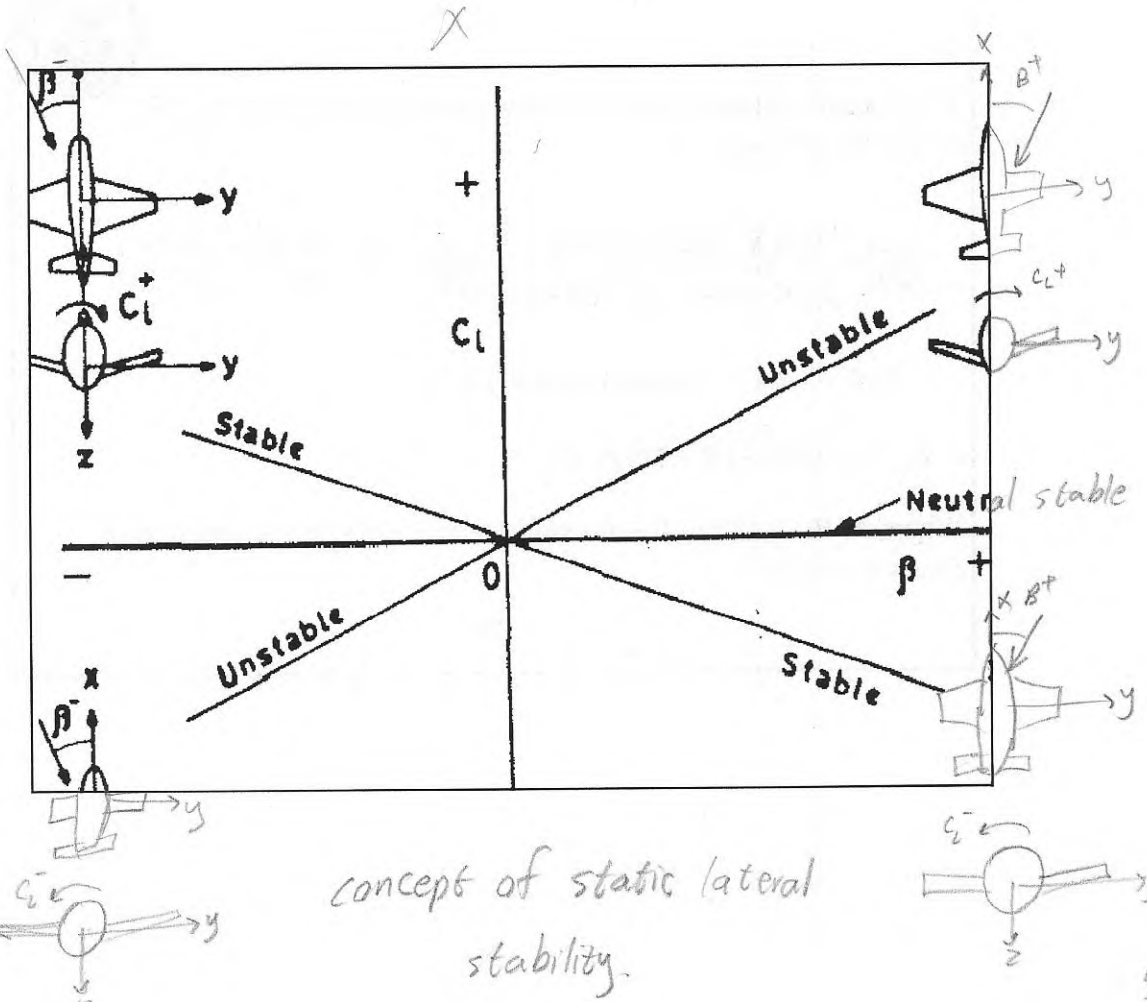
- wing dihedral , Γ
- wing sweep , Λ
- position of the wing on the fuselage
- vertical tail
- thrust effect
- β , α , q

(77 –V)

With regard to the rolling moment



77-A-I



concept of static lateral stability.

An A/C is said to be directionally stable if it has

an inherent capability to realign itself into the resultant wind

whenever disturbed from steady flight .

77-A-II

$$C_\ell = C_{\ell_0} + C_{\ell_\beta}\,\beta + C_{\ell_{\delta_A}}\,\delta_A + C_{\ell_{\delta_R}}\,\delta_R$$

$C_{\ell_0} \approx 0$ except for a/c with long slender fore-bodies, fighter.

C_{ℓ_β} : dihedral effect

$C_{\ell_{\delta_A}}$: lateral control derivative. Imp. in the bank angle maneuver of a/c

$C_{\ell_{\delta_R}}$: Cross control derivative

78-I

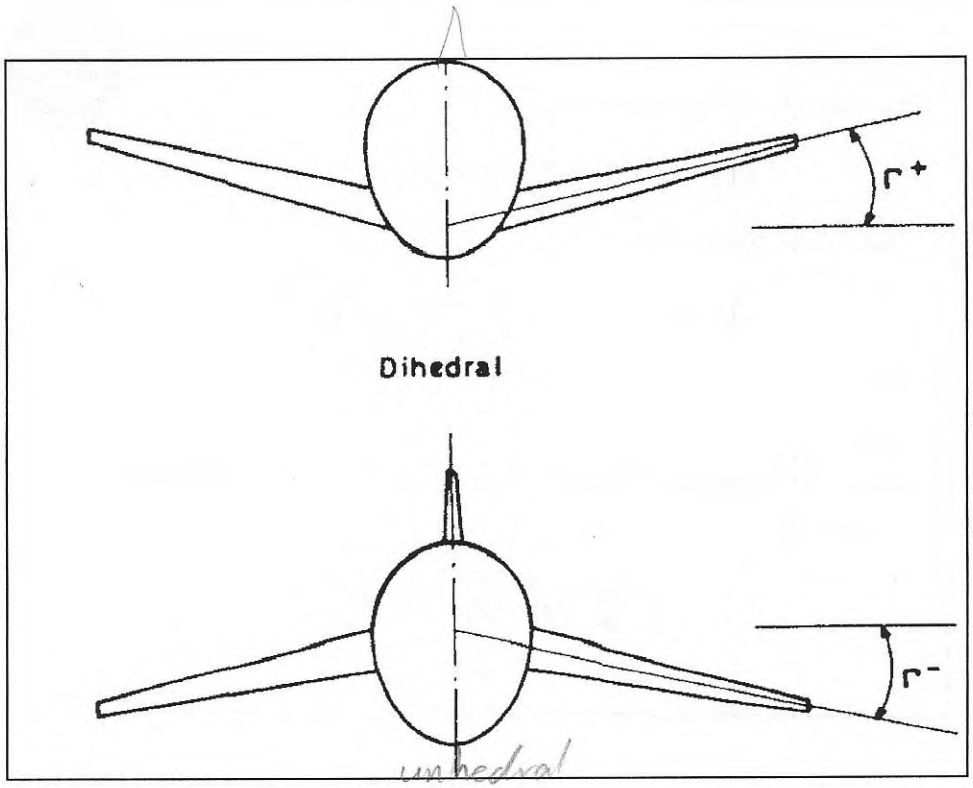
$$C_{\ell_\beta} = C_{\ell_{\beta_{wb}}} + C_{\ell_{\beta_V}} + C_{\ell_{\beta_H}}$$

Very imp in determing a/c stability.

wing body contribution:

Note: primary contribution to C_{ℓ_β} is the
- wing geometric dihedral
- wing position on the fuselage
- wing sweep angle

wing Its dihedral angle, AR an Λ_{LE}



a) wing geometric dihedral:

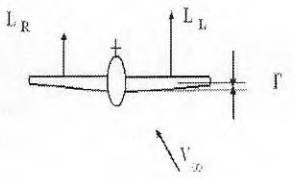
Positive dihedral : wing tip is higher then the root section.

Negative dihedral (unhedral) : wing tip is lower then the root section.

When the a/c is disturbed from a wing level altitude .

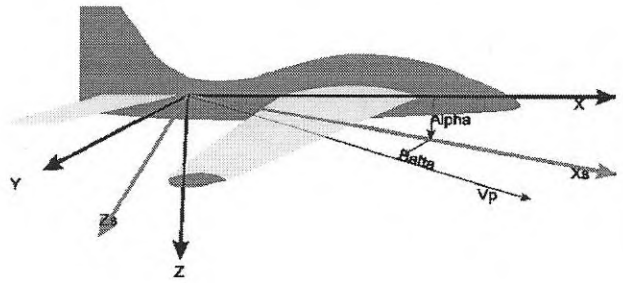
Leading wing experiences an ⬆ in AOA (right wing).

Trailing wing experiences an ⬇ in AOA (left wing).

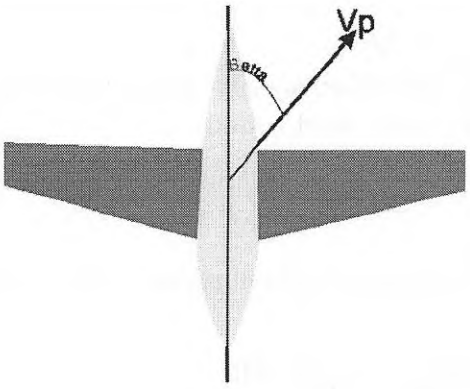


creating a rolling moment to bring the wing back to a wing-level attitude.

78-III

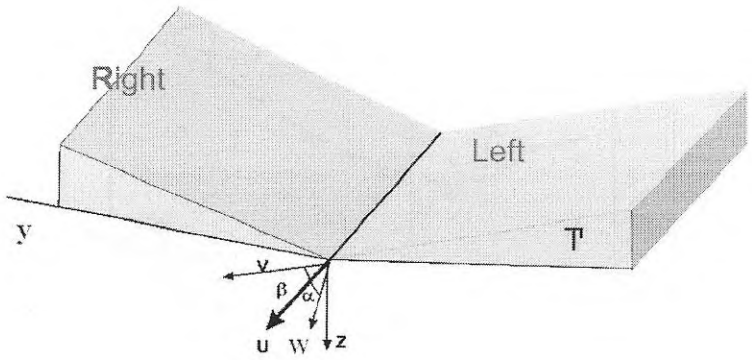


78-A-I



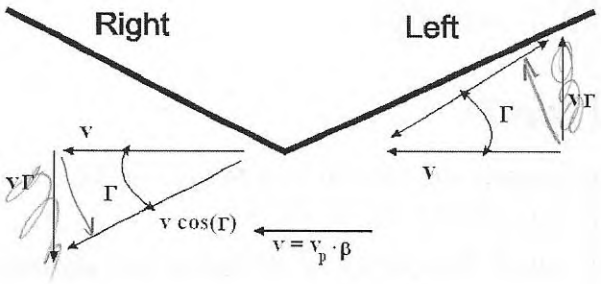
$V_p \cos\beta = u$ $V_p \sin\beta = v$

78-A-II



78-A-III

To the plane of the wing :

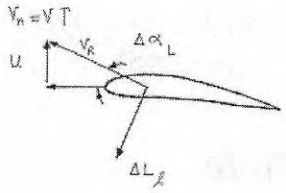


77-A-IV

Now look at the airfoil :

.left airfoil :

$$\Delta\alpha_L = \frac{V_n}{u}$$



V_n : normal comp. of side velocity

u : fwd velocity

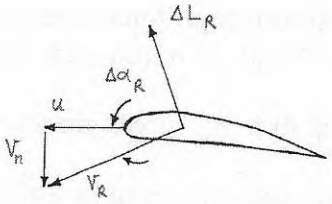
V_R: resultant velocity

78-B-I

.Right airfoil :

$$\Delta\alpha_R = \frac{V_n}{u}$$

U= V_p Cos β ≈ V_p



78-B-II

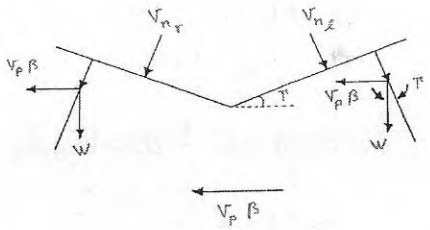
$V_{n,\ell} = W \cos \Gamma - V_p \beta \sin \Gamma$

$\approx W - Vp\, \beta\, \Gamma$

$V_{n,r} = W \cos \Gamma + V_p \beta \sin \Gamma$

$\approx W + Vp \beta\, \Gamma$

$$\Delta\alpha_L = \frac{V_n}{V_p} \; ; \Delta\alpha_R = \frac{V_n}{V_p}$$



$\Delta\alpha_L = (W/Vp) - \beta\, \Gamma$

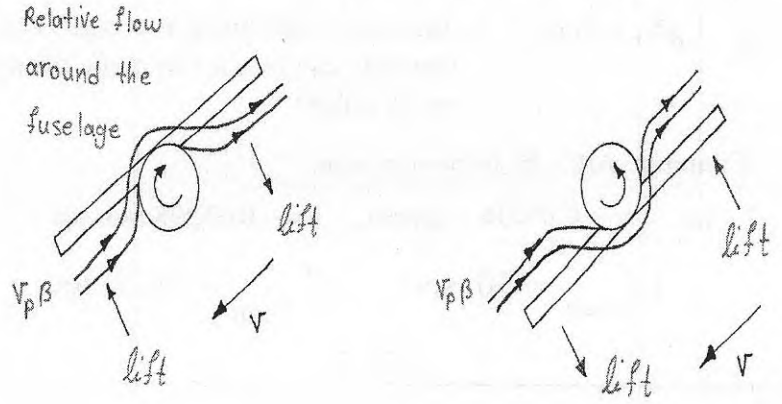
$\Delta\alpha_R = (W/Vp) + \beta\, \Gamma$

$\Delta\alpha = \beta\, \Gamma$: left wing experiences a Neg. ↓ in AOA

78-B-III

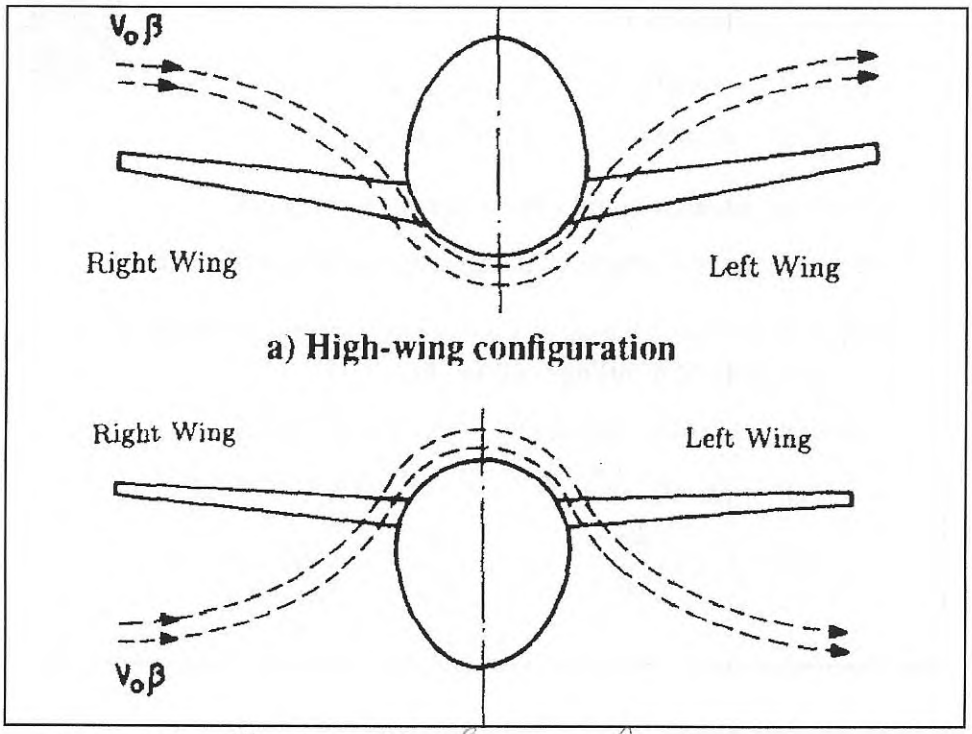
•rolling moment due to sideslip as caused by Γ is ∝ to geometric dihedral angle .

-position of the wing on the fuselage :



79 – I

rolling ناشی از ممان
sideslip به وجود Γ به
می‌آید متناسب با زاویه
dihedral هندسی است



schematic illustration of wing-fuselage interference in slideslip.

For high wing body comb. $C_{\ell_\beta} < 0$

$\Delta C_\ell = 0.0006$

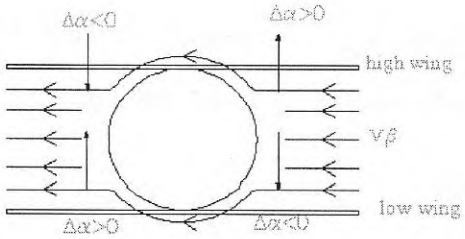
For low wing body comb. $C_{\ell_\beta} > 0$

$\Delta C_\ell = -0.0008$

*most low wing A/C have + dihedral
*most high wing A/C have zero dihedral

79 – II

OR



Body induces vertical velocities then when combined with the main stream velocity, alter the local A.O.A of the wing.

High wing : Negative rolling moment → enhancing dihedral effect.
Low wing : dihedral effect is diminished by the fuselage interference.
(unstable contribution)

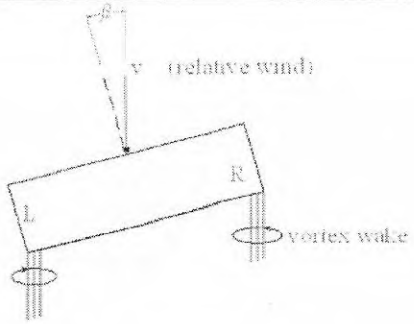
80.I

Can show that for a rectangular wing with C=const. ;

$$C_{l\beta} = -\frac{a_0 \Gamma}{4} \quad (*)$$

.Even if $\Gamma=0$,a flat lifting wing panel has a $C_{l_\beta} \propto C_L$

79.A.I



The vertical induced velocity (downwash) of the vortex wake of L > R by virtue of geometry of the wake →

∴ local wing A.O.A and lift of L< R → negative C_l

Since the induced velocity is ∝ to $C_L \rightarrow \Delta C_{l_\beta} \propto C_L$

79.A.II

(*) : Effect of Γ>0 is stabilizing & is a function Of Γ & a0 (sectional lift carve slope)

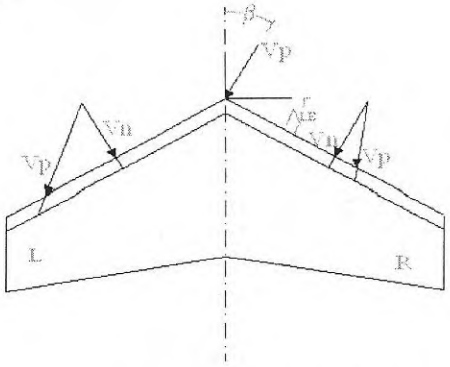
.Excessive Γ , causes discomfort to the passenger since the aircraft will be too sensitive in roll to any atmospheric turbulence or gusts and is disadvantage during crosswind take-off & landing as well as flight with asymm. power.

79.A.III

Magnitude of the dihedral effect depends on:

- Fuselage length ahead of the wing
- Fuselage cross section shape
- Plan form and location of the wing

-Wing Sweep



80.II

Sweepback of the wing is as important as Γ.

$$V_{nR} = V_P . \cos\left(\Lambda_{LE} - \beta\right)$$

$$V_{nL} = V_P . \cos\left(\Lambda_{LE} + \beta\right)$$

$$\Rightarrow q_L < q_R \Rightarrow \quad \Delta L_L < \Delta L_R$$

If each panel were to act as infinite yawed wing at $C_L \rightarrow$

$$\Delta L = C_L . \frac{1}{2} . \rho . V^2 . S\left[\cos^2\left(\Lambda_{LE} - \beta\right) - \cos^2\left(\Lambda_{LE} + \beta\right)\right]$$

80.III

$$\Rightarrow \Delta L = 2.\beta.C_L.q.S.\sin(2\Lambda)$$ For small β

The derivative to rolling moment With $\beta \propto \sin(2\Lambda)$

Thus highly sweep wing tend to have large (-) value of $C_{l_{\beta w}}$ because the side into the wind, gusts higher q.

-For $\Lambda=45^\circ$, $\Delta C_{l\beta} \approx 1/5 C_L$. For large value of C_L , It is a large contribution equivalent to $\Gamma \approx 10^\circ$.

-The effect of Λ various with C_L , becoming vanishingly small at high speed (low C_L) and large at low speed.

- To offset large negative C_{l_β} due to sweep, put geometric anhedral into either the wing or the horizontal tail (F-4 , Harrier jet) $C_{l_\beta} \approx C_L$

81.I

. For Forward swept wings the sign of the ΔL Eq. is reversed → negative sweep is destabilizing.

From the above Eq. it is seen that

.rolling moment due to sideslip, due to sweep (aft) is negative.
.rolling moment due to sideslip, due to sweep (aft) ∝ to C_L .
.rolling moment due to sideslip, due to sweep (aft) ∝ to the sine of twice of the Λ_{LE} .

. C_{l_β} of a/c is of major imp. to stability & controllability & ∝ to C_L → dependence to wing loading & q → significant consequences to configuration Design.

. C_{l_β} more negative → Aircraft is more spirally stable & ↓ the Dutch-roll damping ratios → design conflict.

81.A.I

Can show that for a wing with sweep Λ (back)

$$C_{l\beta} = -\frac{2.a_0.\sin(\Lambda).C_L}{S.b}.\int_0^{\frac{b}{2}\sec\Lambda} C(y_h)\, y_h\, dy_h$$

y_h : spanwise chord.

$C(y_h)$: local chord $\perp$ to the wing L.E.

81.A.II

Horizontal tail contribution

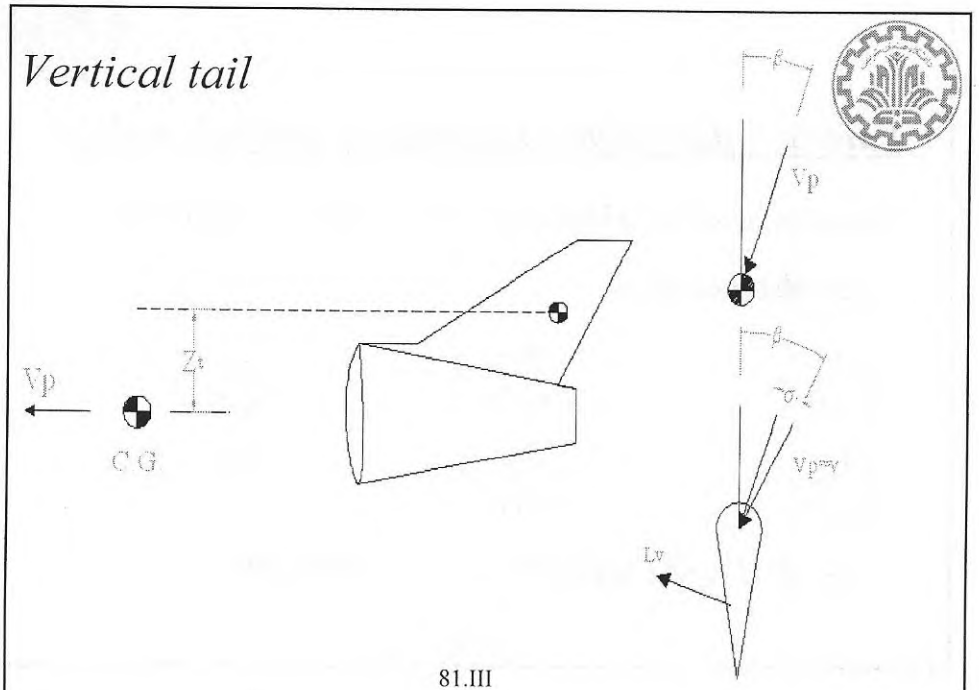
Contribution is the same as wing . Usually small due to it's size.

$$\Delta l_H = \overline{C_{l_{\beta H}}}.\beta.\overline{q_H}.S_H.b_H$$

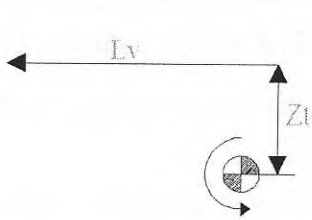
$$C_{l_{\beta H}} = \overline{C_{l_{\beta H}}}.\frac{\overline{q_H}.S_H.b_H}{\overline{q}.S.b}$$ } very small except for large geom. Dihedral, F-4 , general dyn. , F111A , Harrier

81.II

Vertical tail



81.III



If vertical tail Aerodynamic center is above the X-stability axis, a rolling moment due to sideslip is generated.

اگر مرکز آیرودینامیکی دم عمودی بالای محور پایداری X باشد یک میان roll ناشی از sideslip تولید می‌کند.

$$C_{L_v} = C_{L_{\alpha v}}(\beta - \sigma) = C_{L_{\alpha v}}\left(1 - \frac{d\sigma}{d\beta}\right).\beta$$

$$\sigma = \underbrace{\sigma_0}_{\approx 0} + \frac{d\sigma}{d\beta}.\beta$$

82.I

$C_{L_{\alpha V}}$: Lift curve slope of vertical tail → شیب منحنی لیفت بال عمودی

σ:Sidewash induced at the V-T by the wing body contribution.

$$\underbrace{\Delta l_V}_{Rolling-mom.} = -Z_V . C_{L\alpha V} . \left(1 - \frac{d\sigma}{d\beta}\right) . \beta . \overline{q_v} . S_V$$

$$C_{l_{\beta V}} . \beta . \overline{q} . S . b = -Z_V . C_{L\alpha V} . \left(1 - \frac{d\sigma}{d\beta}\right) . \beta . \overline{q_v} . S_v$$

$$C_{l_{\beta V}} = -C_{L_{\alpha V}} . \left(1 - \frac{d\sigma}{d\beta}\right) . \eta_v . \frac{S_V}{S} . \frac{Z_V}{b} \quad ; \quad \eta_V = \frac{V_V{}^2}{V_P{}^2}$$

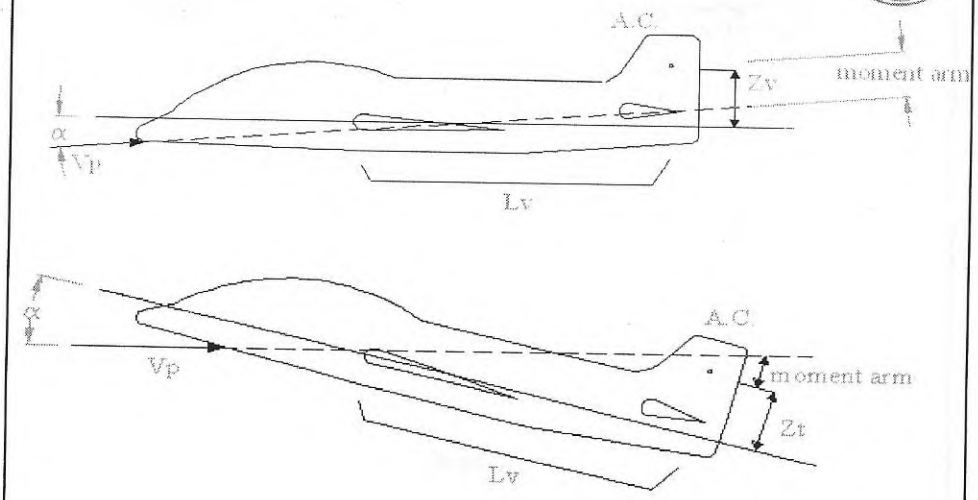
82.II

Its important depends on S_V & Z_V : Z_V depends on α

and at high α may change sing.

$$* \quad C_{l_{\beta V}} = f\left(C_{L_{\alpha V}}, \frac{d\sigma}{d\beta}, Z_V, \frac{S_V}{S}, \frac{Z_V}{b}, AR_V\right)$$

82.III



82.A

Lateral control in Roll

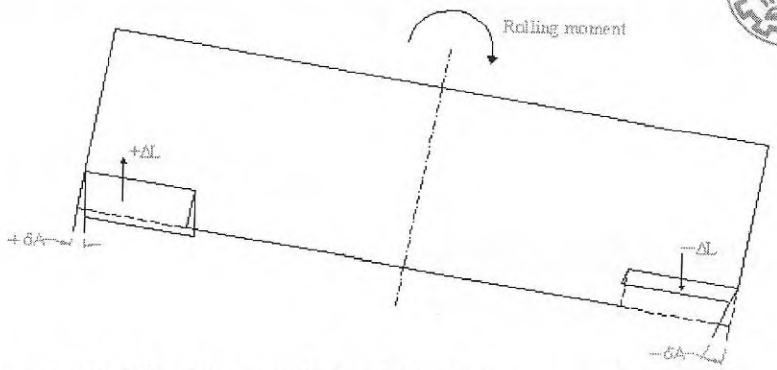
(Roll control Derivatives)

$C_{l_{\delta A}}$ - Lateral control power $\delta A>0$

Right Aileron Up
Left Aileron down $\delta_A = \frac{1}{2}.\left(\delta_{A_R} + \delta_{A_L}\right)$

-Aileron , spoilers , differential stabilizer , etc.
. Also produce yawing moment too.

82.III



Ailerons are good for low speed & relatively low sweep angle applications.

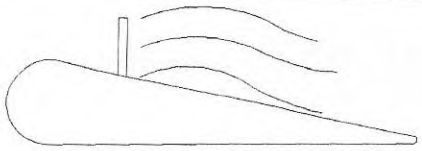
At high speeds, have Aero-elastic effects, no good.

Adverse yaw - $C_{n_{\delta_A}}$

83.I

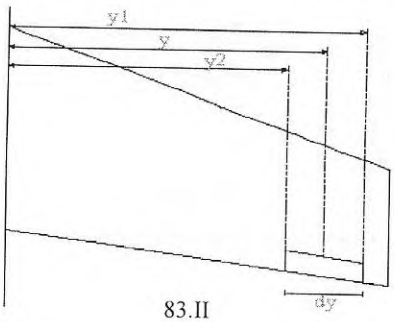
Aileron برای سرعتهای پایین خوب است و نسبتاً زاویه sweep کم استفاده می شود

در سرعتهای بالا ما اثرات آیروالاستیک داریم که خوب نیست.



Spoilers : good at high speed gives good $C_{n_{\delta_A}}$ charact. disadvantage handling quality

To calculate the effect of Aileron, use simple strip integration method.



83.II

Spoilers : برای سرعتهای بالا خوب است که یک $C_{n_{\delta_A}}$ خوبی دهد معایب کیفیت handling

برای محاسبه اثر Aileron از یک نوار باریک روش انتگرالگیری استفاده می کنیم

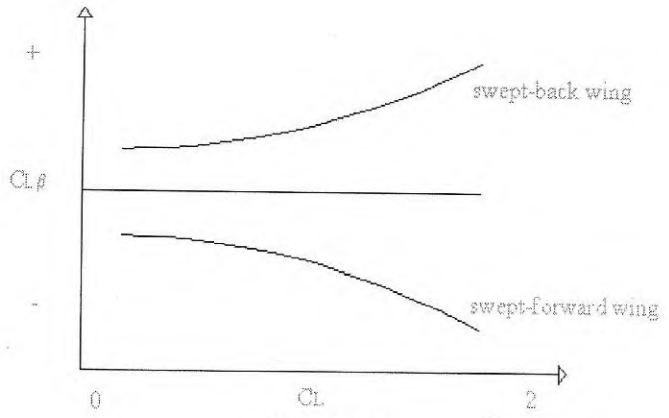
$$C_l = \frac{2.C_{L\alpha W}.\tau.\delta A}{S.b}\int_{y_1}^{y_2} C\ y\ dy$$

$$\tau = \frac{d\alpha}{d\delta_A}\ ;\ C_{l_{\delta_A}} = \frac{2.C_{L\alpha W}.\tau}{S.b}\int_{y_1}^{y_2} C\ y\ dy$$

Note the effectiveness due to stall, swept, Unswept, Fwd swept.

83.III

slipstream effect on C_{l_β} ,flaps down ; series for flap down condition its effect is destabilizing.



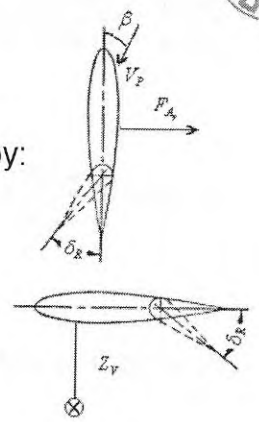
flaps down

83.IV

Cross Control Effect $C_{l_{\delta_R}}$ (rolling moment due to directional control)

Directional control is usually done by:
- Rudder
- Vertical Tail

δ_R - Rudder deflection angle
i_V - Incidence V-T deflection angle



The force due to rudder on V-T

$$F_{Ay} = C_{y_{\delta_R}} \delta_R \bar{q} S$$

84.I

$$l_{rudder} = F_{Ay} Z_V = C_{y_{\delta_R}} \delta_R \bar{q} S Z_V$$

$C_{l_{\delta_R}} = C_{y_{\delta_R}} \frac{Z_{VS}}{b}$: $C_{l_{\delta_R}}$ can reverse the sign due to A. O. A. & location of Z_V

$C_{l_{\delta_R}} = C_{L_{\alpha_V}} \eta_V \frac{S_V}{S} \frac{Z_V}{b} \frac{\partial \alpha_V}{\partial \delta_R}$ Normally > 0 but at high A. O. A. < 0

Size of rudder must be such that to accomplish spin recovery, asymmetric power condition, cross-wind landing, and adverse yaw for transport A/C up to 51 $\frac{ft}{sec}$

84.II

Adverse yaw:

When an A/C is banked, in order to execute a turning maneuver, the aileron may create a yawing moment that opposes the turn (adverse yaw). The rudder must be able to overcome the adverse yaw. So that a coordinated turn be achieved. Critical condition for adverse yaw occurs when flying at low speed (high C_L)

84.III

IN THE NAME OF GOD

Flight Mechanics II

Side force: $F_{Ay} = C_y \bar{q} S$

$$C_y = C_{y_0} + C_{y_\beta}\beta + C_{y_{\delta_R}}\delta_R + C_{y_{\delta_A}}\delta_A$$

Some books write $C_y = C_{y_0} + C_{y_\beta}\beta + C_{y_{\delta_R}}\delta_R + C_{y_{\delta_A}}\delta_A + C_L\phi$

ϕ - bank angle

-can be generated by many parts of the A/C; Main contributors are

- vertical tail
- An inclination of the lift vector.

85.I

- If side force is generated in a sideslip, it creates the roll-yaw coupling problem which makes the piloting task difficult.

- Considerable work is being done on "direct side force control" thru auxiliary surfaces on wing or fuselage.

- Inclination of lift vector has great influence on the side force.

$$C_{y_\beta} = C_{y_{\beta WB}} + C_{y_{\beta V}}$$

85.II

C_{y_β} important in Dutch-roll dynamic.

$C_{y_{\beta WB}}$ Generally small compared to $C_{y_{\beta V}}$:

It is important when $\Gamma \neq 0$

usually $C_{y_{\beta WB}} < 0$, the fuselage contribution depend on:

- Placement of wing on fuselage
- Size of fuselage in relation to the wing

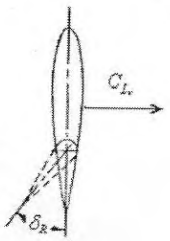
$$C_{y_{\beta WB}} = -0.0001|\Gamma|57.3/rad$$

85.III

Vertical tail : $C_{y_{\beta_V}}$

$$C_{L_V} = C_{L_{\alpha_V}}(1-\frac{d\sigma}{d\beta})\beta$$

$$C_{y_{\beta_V}} = -C_{L_{\alpha V}}(1-\frac{d\sigma}{d\beta})\eta_V \frac{S_V}{S} = f(AR_V, \Lambda_V, ..)$$



Twin vertical tail:

$$C_{y_{\beta_V}} = -2\left\{\frac{C_{y_{\beta_V(WBH)}}}{C_{y_{\beta_V eff}}}\right\} C_{y_{\beta_V eff}} \frac{S_V}{S} \ /rad$$

S_V – Area of a single vertical tail panel

$C_{y_{\beta_V(WBH)}}$: Wing body horizontal tail interference

$C_{y_{\beta_V eff}} - f(A_{eff})$ 86.I

Control derivative: $C_{y_{\delta_R}}$

$$C_{L_V} = \frac{\partial C_L}{\partial \delta_R}\delta_R = C_{L_{\alpha V}}\frac{\partial \alpha_V}{\partial \delta_R}\delta_R = C_{L_{\alpha V}}\alpha_{\delta_R}\delta_R = C_{L_{\alpha V}}\tau_R\delta_R$$

τ_R -Change in A.O.A with vertical rudder deflection.

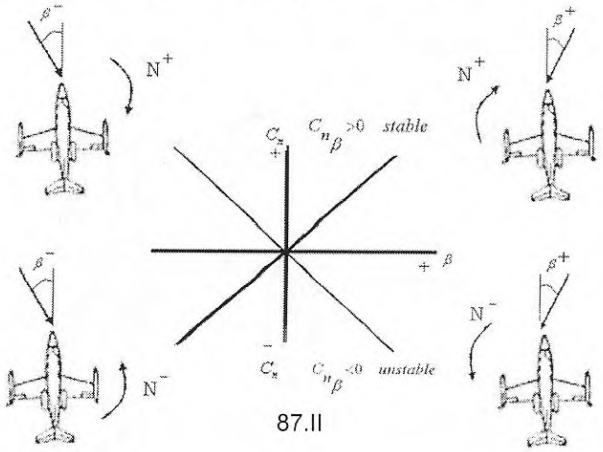
$$C_{y_{\delta_R}} = C_{L_{\alpha V}}\tau_R\eta_V\frac{S_V}{S}$$

$$C_{y_{\delta_A}} \cong 0 \Rightarrow C_y = C_{y_\beta}\beta + C_{y_{\delta_R}}\delta_R$$

86.III

Yawing moment $N_A = C_n \bar{q} S b$

$$C_n = C_{n_0} + C_{n_\beta}\beta + C_{n_{\delta_R}}\delta_R + C_{n_{\delta_A}}\delta_A$$



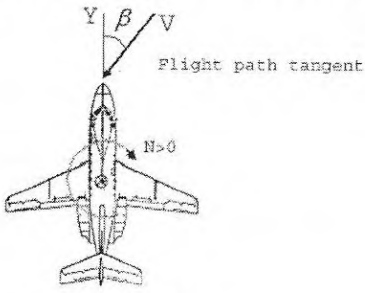
87.II

stiffness
توانایی یک سیستم برای مقاومت در برابر انحراف تعیین شده.

dutch-roll:
نوسانات مرضی هواپیما با حرکتی از چرخش های حول محور طولی و محور عمودی این حالت در swept wing ها می شود

$C_{n_0} = 0$ if A/C is aerodynamically symmetric

$C_{n_\beta} > 0$ Yaw stiffness or weathercock stability
very important in dutch roll & spiral dynamic



87.III

برای داشتن پایداری جهتی استاتیکی هواپیما باید یک ممان yaw ایجاد کند که هواپیما را به پایداری تعادلیش برساند

یک هواپیما که پایداری جهتی استاتیکی داشته باشد همیشه نشان به سوی بادنسبی دارد و بنابراین نام آن را پایداری بادنما می گویند.

To have static directional stability, the A/C must develop a yawing moment which will restore the A/C to its equilibrium stability.

An A/C possesses static directional stability will always point into the relative wind, thus the name "weathercock stability".

$$C_{n_\beta} = C_{n_{\beta_{WB}}} + C_{n_{\beta_V}} = C_{n_{\beta_W}} + C_{n_{\beta_B}} + C_{n_{\beta_V}} :$$

$$C_{n_{\beta_W}} = (C_{n_\beta})_{\Gamma,W} + (C_{n_\beta})_{\Lambda,W}$$

87.IV

$$(C_{n_\beta})_{\Gamma,W} \cong 0.075\,\Gamma\,C_L \quad /rad \qquad :\Gamma : Radian$$

در سرعت supersonic سهم کوچک است و می توان نادیده گرفت.

For subsonic speed,

For supersonic speed the contribution is small and can be ignored.

$$\frac{(C_{n_\beta})_{\Lambda,W}}{{C_L}^2} = \frac{1}{4\pi\,AR} - \frac{\tan\Lambda_{c/4}}{\pi\,AR\,(AR + 4\cos\Lambda_{c/4})}\left[\cos\Lambda_{c/4} - \frac{AR}{2} - \frac{AR^2}{8\cos\Lambda_{c/4}} - 6\bar{X}_a\frac{\sin\Lambda_{c/4}}{AR}\right] \quad rad$$

$\Lambda_{c/4}$: wing quarter chord sweep → وتر بال $\frac{1}{4}$ sweep

فاصله میان cg و ac بال در ترم میانگین آیرودینامیکی chord در بال.

$\bar{X}_a$: distance between the c.g. & the wing a.c. in term of mean aerodynamic chord of the wing.

اگر cg عقب مرکز آیرودینامیکی بال باشد کوپرایی سرعت supersonic بالاست

& $\bar{X}_a > 0$ if c.g. is aft. of wing aerodynamic center the above are for subsonic speed.

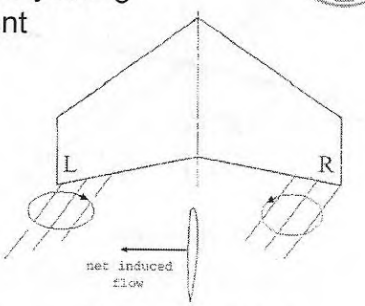
87b.I

$C_{n_{\beta W}}$ is small except at H. A. O. A. & is negative.

$q_R > q_L \quad \therefore D_R > D_L \Rightarrow$ Positive yawing moment

$C_{n_{\beta B}} < 0$ usually

$C_{n_{\beta Wf}} = -k_n \, k_{R_l} \frac{S_{fs}}{S_W} \frac{l_f}{b}$ For subsonic



Side wash is + from left to right

K_n - An empirical wing body interference factor and is a function of fuselage geometry.

K_{R_l} - An empirical correction factor and is a function of the $(R_e)_f$

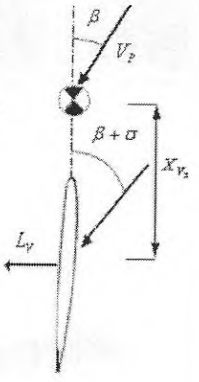
88.I

S_{f_s} - projected side area of the fuselage

l_f - length of the fuselage

Since W+F contribution is destabilizing

$\therefore$ A vertical tail must be properly sized to ensure directional stability.



$\alpha_V = \beta - \sigma = (1 - \frac{d\sigma}{d\beta})\beta$ Some books $\alpha_V = \beta + \sigma$

$N_V = C_{L_{\alpha_V}} (1 - \frac{d\sigma}{d\beta}) \beta \, \bar{q}_V \; S_V \, X_{V_S} = C_{n_N} q_V S b = C_{n_{\beta_V}} \beta \bar{q} S b$

$C_{n_{\beta_V}} = C_{L_{\alpha_V}} (1 - \frac{d\sigma}{d\beta}) \frac{\bar{q}_V}{q} \frac{S_V}{S} \frac{X_{V_S}}{b}$

88.IV

نسبت حجم دم عمودی برای sizing اولیه بسیار مهم است.

$V_V = \frac{S_V}{S}\frac{X_{V_S}}{b}$ Vertical tail volume ratio.
Very important for initial sizing.

$$\therefore C_{n_{\beta_V}} = V_V \eta_V C_{L_{\alpha_V}} (1 - \frac{d\sigma}{d\beta})$$

$$(1 + \frac{d\sigma}{d\beta})\eta_V = 0.724 + 3.06 \frac{s_V / S}{1 + \cos \Delta_{c/4_W}} + 0.4 \frac{z_W}{d} + 0.009\, AR_W$$

S - Wing area d - max. fuselage depth

S_V - Vertical tail area including the submerged area to fuselage centerline

Z_W - Distance from the wing root quarter chord pt. to the fuselage centerline

AR_W - Aspect ratio of wing

89.II

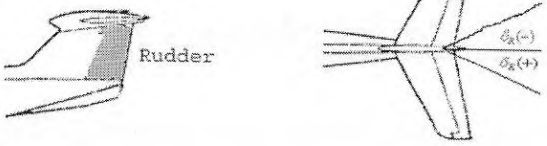
in flight testing $C_{L_{\alpha_V}}$ can be changed by adding:

- Dorsal fin $\Rightarrow$ $\downarrow AR_V$ $\Rightarrow$ Stall at higher β
- Ventral fin $\Rightarrow \uparrow AR_V$ & lower the c.p. of V-T.
  ($\downarrow$ roll with yaw)



89.III

Control for yaw:



$C_{n_{\delta_R}}$: Rudder power or rudder control derivative

$C_{n_{\delta_A}}$: Aileron adverse yaw (adverse aileron yaw effect)

Plays a nuisance role $\cong 0$ or $\begin{matrix}<0\\>0\end{matrix}$

Size of rudder is determined by the directional control requirement.

$C_{n_{\delta_R}}$ is important in coordinating turn & asymmetric thrust.

89.V

$N=-Y_V * X_V$ $\quad Y \rightarrow$ Side force

$$Cn=-\frac{q_V}{q}\frac{S_V}{S}\frac{X_V}{b}\frac{dC_{L_V}}{d\delta_R}\delta_R$$

Left deflection of rudder is +

$$Cn=-\eta_V V_V \frac{dC_{L_V}}{d\delta_R}\delta_R$$

Or

$$Cn=Cn_{\delta_R}\delta_R=-\eta_V * V_V \frac{dC_{L_V}}{d\delta_R}\delta_R$$

$$Cn_{\delta_R}=-\eta_V V_V \frac{dC_{L_V}}{d\delta_R} \qquad \frac{dC_{L_V}}{d\delta_R}=\frac{dC_{L_V}}{d\alpha_V}\frac{d\alpha_V}{d\delta_R}=C_{L_{\alpha_V}}\tau$$

(90)

$$\therefore Cn_{\delta_R} = -C_{L_{\alpha_V}} \alpha_{\delta_R} \eta_V V_V$$

Cn_{δ_A} : Aileron yawing mom coff. Derivative ; Varies from A/C to A/C ,

Cn_{δ_A} Has positive deflection (R up, L down)

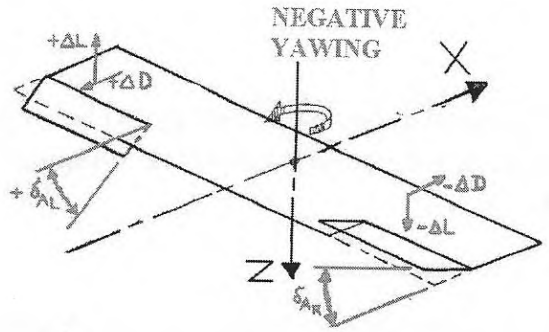
$\therefore \uparrow$ Lift on the left wing, $\downarrow$ lift on the right one.

Also D on left wing $\uparrow$ & $\downarrow$ on right wing

$\Rightarrow$ Rolling to the right

(90 II)

Negative yawing mom. Due to aileron deflection



$\uparrow$ Lift $\Rightarrow \uparrow$ Di & $\downarrow$ lift on R.h.s $\Rightarrow \downarrow$ Di

$\Rightarrow$ Neg. yawing mom , called adverse yaw & tends to yaw an A/C out of an intended turn

Note the profile drag for both ailerons are the same $\Rightarrow$ no yawing mom.

(90 A II)

Frise aileron:

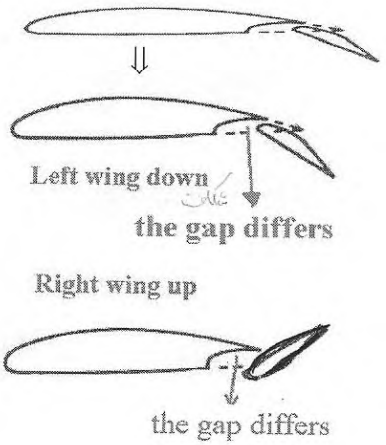
aileron که لبه حمله آن به صورت شیب دار پهن و از خط لولا جلوتر می آید. پایین آمدن این aileron سبب ایجاد یا شتاب سطح مؤثر بال می گردد ولی بالا آمدن آن سبب پایین تر رفتن لبه حمله aileron از سطح تحتانی بال، افزایش پسا و برابر کردن پسای aileron هنگام دور زدن می گردد.

حذف کردن

To eliminate neg. yaw due to aileron deflection

I) Use Fries aileron:

Frise



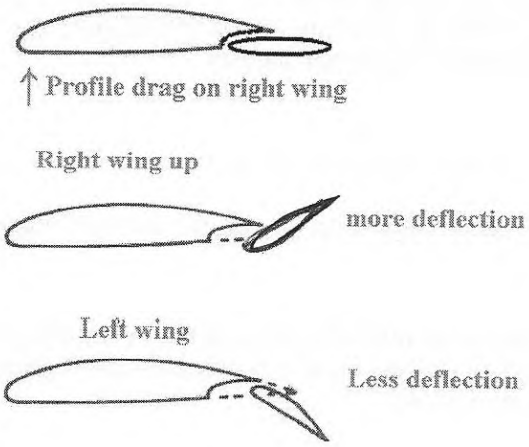
&

(90 A III)

Differential ailerons:

ارتباط aileron ها به قسمی که زاویه حمله aileron که به طرف بالا حرکت می کند از زاویه حرکت aileron دیگر که به طرف پایین حرکت می کند، بیشتر است. این وضعیت سبب می شود که پسای بال با aileron بالا رونده بیش از نیروی پسای بال با aileron پایین رونده افزایش یابد.

ii) Differentially deflected aileron:



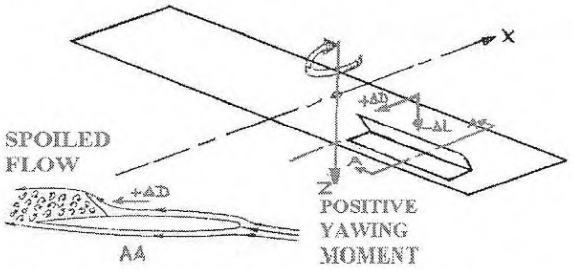
(90 B)

.Spoiler yawing mom. Coff. Derive, $C_{n_{\delta_s}}$:

Can use it to produce + yawing mom

referred to proverse yaw & is preferred over

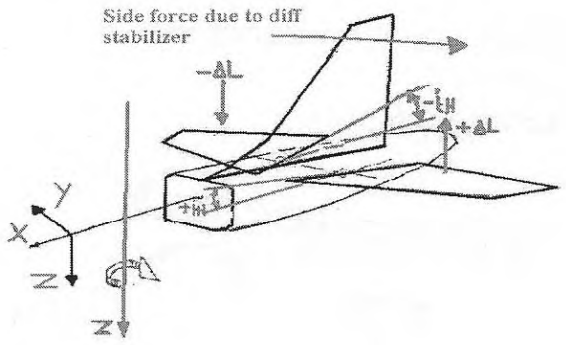
adverse yaw unless it become too proverse



↓ Lift on R.h.s ⇒↓Di , but flow Sep.⇒↑ in profile drag

(90 B I)

. Differential stabilizer yawing mom. Coff deriv. $C_{n_{i_h}}$:



(90 B II)

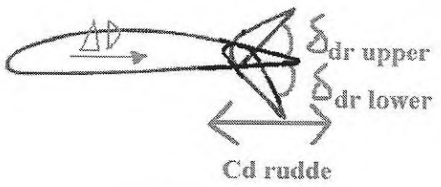
Since the C.G of the A/C is usually fwd. of the vertical tail

→ A yawing momentum due to diff. stabilizer deflection will be generated

If significant, must run wind tunnel test to establish its mag.

Drag rudder yawing mom. coff. deriv.

$C_{n_{\delta_{R_{drag}}}}$:



(90 C)

$$N_{A_{dr}} = \Delta D_{dr}\, Y_{dr}$$

Y_{dr}: Distance between the drag rudder C.P & the C.G measurement along the stability axis

$$\Delta D_{dr} = \frac{\partial C_{D_{dr}}}{\partial \delta_{dr}} \delta_{r\,drag}\, \bar{q} S$$

$$\delta_{r\,drag} = \frac{1}{2}(\delta_{r\,drag_{upp}} + \delta_{r\,drag_{lower}})$$

$$\frac{\partial C_{D_{dr}}}{\partial \delta_{r\,drag}} \cong 0.025 \frac{b_{dr}}{S} C_{dr} \quad /\text{deg}$$

(90 C I)

b_{dr} : Span of the drag rudder

C_{dr} : Chord of the drag rudder

$$\Rightarrow Cn_{\delta_{r\,drag}} \cong 0.025 \frac{b_{dr} \cdot C_{dr}}{S \cdot b} Y_{dr}$$

(90 C II)

For typical A/C config, good handling qualities are found with $0.06 \leq C_{n_\beta} \leq 0.15$ /rad & $C_{n_\beta} \cong .03$ is the lower limit

If A/C has high wing loading & low span C_{n_β} must be ↑.

Usually the more static stability an A/C has in yaw, the better it will handle

For further info & detailed one refer the CHP 5 of Phillips.

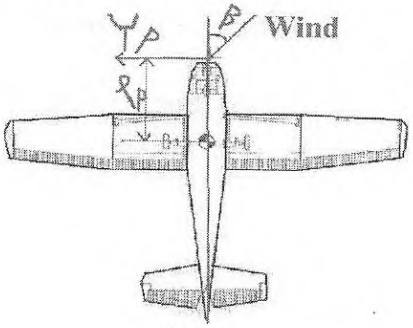
(90 D)

Propeller effect:

A)Power effect:

$$Cn_{\beta} = \frac{Y_p * l_p}{q\ s\ b}$$

Y : Cross wind force



(90 A)

$$Cn_{\beta}\Big|_p = \frac{\pi D^2 l_p (\frac{dC_{y_p}}{d\beta}) N}{q\ S_w\ b}$$

N→no. of propeller

$$\frac{dC_{y_p}}{d\beta}\Big|_{TC=0}$$

2 blade propeller = 0.00165
3 blade propeller = 0.00235
4 blade propeller = 0.00296
6 blade propeller = 0.00510

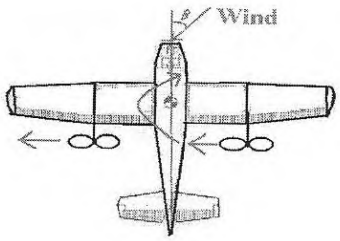
With full power $C_{n_\beta}\Big|_{Full\ Power} \cong 1.5 C_{n_\beta}\Big|_p$, propeller wind milling

For more info refer to Phillips.

(90 A I)

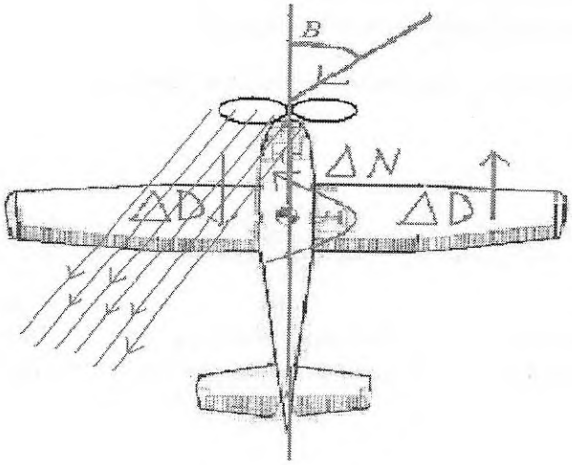
Yawing mom due to propeller:

b) Power effect for pusher A/C:



(90 E)

C) Propeller slipstream effect :



(90 E I)

Lateral stability :

$$-mg\sin\phi\cos\gamma=(Cy_{\beta}\beta+Cy_{\delta_A}\delta_A+Cy_{\delta_R}\delta_R)\bar{q}S+Fy_T$$

$$0=(C_{l_{\beta}}\beta+C_{l_{\delta_A}}\delta_A+C_{l_{\delta_R}}\delta_R)\bar{q}Sb+L_T$$

$$0=(C_{n_{\beta}}\beta+C_{n_{\delta_A}}\delta_A+C_{n_{\delta_R}}\delta_R)\bar{q}Sb+N_T$$

Unknown ϕ , β , γ , δ_A , δ_R , $\bar{q}$, T

But γ , $\bar{q}$, T are known from longitudinal

$\therefore$ Unknown ϕ , β , δ_A , δ_R $\Rightarrow$ 4 unknowns, 3 Eq.

Select one $(\phi$, β , δ_A , $\delta_R)$ and solve for other 3

(91)

preselect ϕ from flying quality req. with engine out.

-Military specific for 1 engine out :

$\Rightarrow$

-Straight line flight

$|\phi| \le 5°$

-For all speed $>1.2V_{stall_{T.O}}$

-Civil A/C, F. A. A:

$\Rightarrow$

-Straight line flight

-Speed $>1.2V_{stall_{T.O}}$

- $|\phi| \le 5°$ Or $\beta = 0$

(91 I)

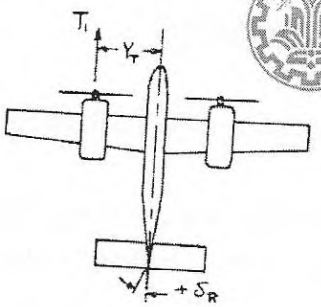
$$\begin{vmatrix} C_{y_\beta} & C_{y_{\delta_A}} & C_{y_{\delta_R}} \\ C_{l_\beta} & C_{l_{\delta_A}} & C_{l_{\delta_R}} \\ C_{n_\beta} & C_{n_{\delta_A}} & C_{n_{\delta_R}} \end{vmatrix} \begin{vmatrix} \beta \\ \delta_A \\ \delta_R \end{vmatrix} = \begin{vmatrix} \dfrac{-mg\cos\gamma\sin\phi + F_{y_T}}{\bar{q}s} \\ \dfrac{-L_T}{\bar{q}sb} \\ \dfrac{-N_T}{\bar{q}sb} \end{vmatrix}$$

Can solve for δ_A , β and δ_R

To get a feeling about control derivative; look at engine out case

92-I

$$N_T = T.y_1 + \Delta N_D$$



ΔN_D - yawing momentum due to drag in engine out, can be as large as N_T

92-II

ΔN_D is due to:

- -drag due to wind milling engine
- -drag due to propeller
- -drag due to feathered Propeller ; an extension of the blade angle to reduce drag when an engine is stopped in flight
- -drag due to rudder deflection
- -spillage drag (Sup.)

92-III

Approximate Analysis:

δ_R =?

Rudder takes care of yawing momentum

92-IV

$\Rightarrow$

$$C_{nT} = T\, y_1 / qSb \quad : \quad qs = \frac{W}{C_L} \quad \text{for} \quad L = W$$

$C_{NT} = C_L . T\, y_1 / Wb$ can be used directly if a/c has jet engines

For propeller driven A/C:

$T = 550 \eta_P \, BHP / V_P$

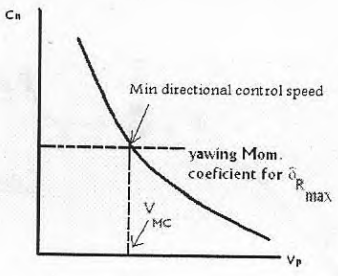
η_P - Propeller efficiency

BHP-brake horse power

$\Rightarrow \; C_{n_T} = 550 \, \eta_P BHP \; C_L . y_1 / V_P Wb$

If hold horsepower & η_P constant, can plot C_{n_T} vs. V_P

92-A-I



V_{MC} is determined by flight test

$V_{MC} \uparrow$ as W $\downarrow$ and also as c.g moves aft.

92-A-II

تقریبی

Approximate Analysis:

δ_R=? Rudder takes care of yawing moment

$$\Rightarrow \quad C_{n_\beta} + \underbrace{C_{n_{\delta_A}} \delta_A}_{\approx 0} + C_{n_{\delta_R}} \delta_R = -\frac{N_T + \Delta N_{D_1}}{\bar{q} s b}$$

Solve for δ_R :

$$\Rightarrow \quad \delta_R = \left(\left[-\frac{N_T + \Delta N_D}{\bar{q} s b} - C_{n_\beta} \beta \right] \Big/ \; C_{n_{\delta_R}} \right)$$

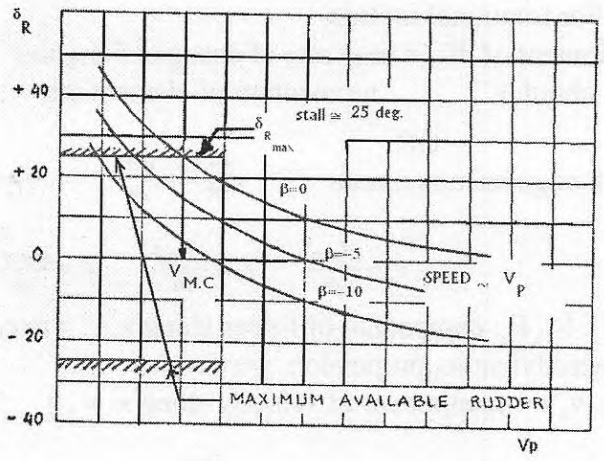
93-I

For zero sideslip: $\beta = 0$

$$\delta_R = -\frac{N_T + \Delta N_D}{\bar{q} s b C_{n_{\delta_R}}} \quad : \quad \bar{q} = \frac{1}{2} \rho \bar{V}_P^2$$

As V ↓ , δ_R ↑

V_{MC} - minimum control speed; cannot fly below V_{MC}



93-II

نیاز به راضی کردن معیارهای بالا داریم. نکته: خلبان می‌تواند V_{MC} کمتر از T.O تجاوز نکند some margin به وسیله

$V_{MC} \leq 1.2\, V_{s_{T.O}}$ { Require to satisfy above criteria. Note: pilot cannot Take-off until V_{MC} is exceeded by some margin.

If want to move the curve we must change $C_{n_{\delta_R}}$

$$\delta_{R_{\max}} = -\frac{N_T}{\frac{1}{2}\rho V_{MC}^2 C_{n_{\delta_R}} Sb} \Rightarrow \begin{cases} V_{MC} = \sqrt{\dfrac{2N_T}{\rho Sb C_{n_{\delta_R}} \delta_{R_{\max}}}} \\ to \downarrow V_{MC} \begin{cases} \bullet\ reduce\ |N_T| \\ \bullet \uparrow \left| C_{n_{\delta_R}} \delta_{R_{\max}} \right| \end{cases} \end{cases}$$

$V_{MC} \leq$ *highest of* $1.1 V_{st}$ (FAR25) or $V_s + 10$ *knots*(*Mil.*)

93-VI

برای یک ترکیب سرعت T.O، تراست موتور و ممان yaw موتور، توانایی کنترل موتور خارج از پرواز بستگی به بزرگی انحراف مثبت کنترل $C_{n_{\delta_R}}$ دارد. rudder

For a given combination of take-off speed, engine thrust & engine Yaw. moment, the ability to control engine out flight depends on magnitude of rudder control power derivative, $C_{n_{\delta_R}}$

$C_{n_{\delta R}} \approx -.001$ typically

93-V

توجه کنید که کوشش پرواز در یک هواپیما چندموتوره در $V < V_{MC}$ یک درخواست برای آزاردادن است.



- Note to attempt flight in a multi-engine a/c at $V < V_{MC}$ is asking for trouble!

توجه کنید این می گیرد ۱ یا ۲ ثانیه برای تحقق بخشیدن (پی بردن به موتور خارج شده) و فعال کردن rudder هواپیما ممکن است شروع به yaw کند و ناشی از اثر dihedral این ممکن است که یک roll شروع کند، سپس

- Note it takes, one or two seconds to realize (find out which engine) and act the rudder ⇒a/c may begin to yaw and due to dihedral effect it may begin to roll, then

$$C_{n_\beta}\beta + \underbrace{C_{n_{\delta_A}}\delta_A}_{\approx 0} + \underbrace{C_{n_{\delta_R}}\delta_R}_{\approx 0} = -\frac{N_T + \Delta N_D}{\bar{q}Sb}$$

The $\beta_{\max}$ is: $$\beta_{\max} = -\frac{\bar{N}_{T_1} + \Delta N_{D_1}}{C_{n_\beta}\,\bar{q}_1 sb}$$

93-A-I



& to keep the wings level, $\delta_{A_{req}}$ is:

$$\delta_{A_{req.}} = -\frac{C_{l_\beta}}{C_{l_{\delta_A}}}\beta_{\max} - \frac{L_{T_1}}{\bar{q}_1 sbC_{l_{\delta_A}}} = \frac{\frac{C_{l_\beta}}{C_{n_\beta}}(N_{T_1} + \Delta N_{D_1}) - L_{T_1}}{C_{l_{\delta_A}}\,\bar{q}_1 sb}$$

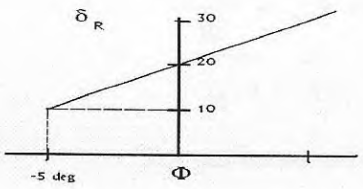
انحراف aileron نباید تجاوز کند بعد از اینکه بال stall کند تقریباً $\leq 25°$

- This aileron deflection should not exceed, since one wing stalls, typically $\leq 25°$

مقدار rudder برای نگه داشتن حالت موتور خارج شده می تواند کاهش دهد به وسیله اجازه دادن به هواپیما که bank بزند به سوی موتور که کار می کند.

- The amount of rudder to hold engine- out condition can be ↓ by allowing the a/c to bank into the operating engine.

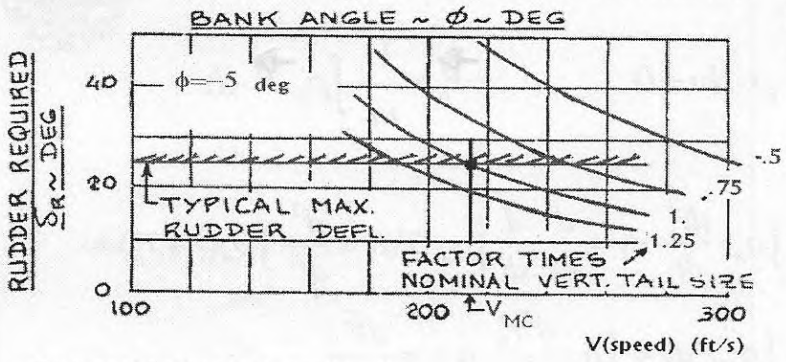
93-A-II



Right engine out

93-A-III

•Size of V-T has large effect on this control problem too



•Rudder deflection to generate β

$$(C_{n_\beta})\beta + C_{n_{\delta_R}}\delta_R = 0 \quad \Rightarrow \quad \delta_R = -\frac{(C_{n_\beta})}{C_{n_{\delta_R}}}\beta$$

$\therefore$ The higher the level of static directional stability, the higher the rudder deflection to generate the β .

بالاترین سطح پایداری جهتی استاتیکی بالاترین انحراف rudder بوسیله β تولید می‌کند.

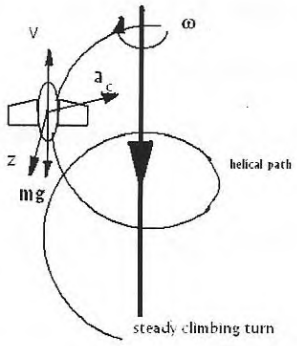
93-B-I

•Steady state turn (angular velocity vector is vertical)

•Steady state symmetrical pull-up

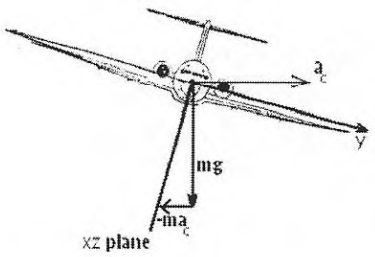
The truly banked turn is the one that:

I)The vehicle angular velocity vector ω is constant.



93-C-I

II) The resultant of gravity & C.F. at the mass center lies in the plane of Symmetry.



93-C-II

این مشابه پرواز چرخشی در مقایس چرخش bank،

چرخشها می‌تواند در زاویهٔ bank بزرگ که خیلی بزرگ برای خطی سازی $\sin\phi$ و $\cos\phi$ هستند اگرچه همه تغییرهای حالت دیگر بجز ϕ و V کوچک هستند.

← مجبوریم از معادلات غیرخطی استفاده کنیم



This corresponds to flying the turn on the turn-bank indicator.

- Turns can be at large bank angle that are too large for linearization of $\sin\phi$ & $\cos\phi$ although all the state variables other than ϕ & V are small.

$\Rightarrow$ have to use non-linear equations.

93-C-III

زاویهٔ bank بزرگ ← ترکیبی از معادلات طولی و عرضی. (لیفت بیشتری برای بالانس گرانش که در سطح پرواز است نیاز است)



Large bank angle $\Rightarrow$ coupling of the lateral & longitudinal equations. (more lift is needed to balance gravity than in level flight)

$\therefore$ For turning at large ϕ, must use aileron, rudder & elevator

The body axis angular rates are:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\theta \\ \sin\phi \\ \cos\phi \end{bmatrix} \omega$$

For small elevator angels, θ.

93-D

Steady State Turns:

Talked about S.S. straight line flight
- S.S turning flight → Angular velocity vector is vertical
- S.S symmetrical pull-up

$\overline{\omega^{\cdot}} = \overline{V^{\cdot}_{P}} = 0$, also $\overline{\omega} = k_1 \psi^{\cdot}$

($\psi^{\cdot}$ - rate of change of azimuth of the A/C x-axis)
Some look
$\psi^{\cdot} = \Omega$ rate of turn; Ω is positive for a right turn

In level turn $\gamma_1 = 0$ & $\theta_1 = 0$ or small
(θ & φ both are constant during maneuver.)
In stability Axis $W_1 = 0$, no component in the Z-axis

94-II

From kinetic equation :

$$\begin{cases} P_1 = -\psi_1^{\cdot} \sin\theta \cong 0 & \text{rolling} \\ Q_1 = \psi_1^{\cdot} \cos\theta_1 \sin\varphi = \psi_1^{\cdot} \sin\varphi & \text{pitching} \\ R_1 = \psi_1^{\cdot} \cos\theta \cos\varphi = \psi_1^{\cdot} \cos\varphi & \text{Yawing} \end{cases}$$

Even for a rapid turn 360°/min , $\psi_1^{\cdot}$ is small $= 2\pi/60 = 0.1$

Force equation

$0 = F_{Ax_1} + F_{Tx_1}$ → 0

$mu_1 R_1 = mg \sin\varphi_1 + F_{Ax_1} + F_{Ty_1}$ → 0

$-mu_1 Q_1 = mg \cos\varphi_1 + F_{Az_1} + F_{Tz_1}$

Assume thrust only contribute to X-direction

94-III

Moment. Equation :

$$\begin{cases} (I_{zz} - I_{yy}).R_1 Q_1 = L_{A1} + L_{T1} \\ (I_{xx} - I_{zz}).P_1 R_1 + I_{xz}.(P_1^2 - R_1^2) = M_{A1} + M_{T1} \\ (I_{yy} - I_{xx}).P_1 Q_1 + I_{xz} Q_1 R_1 = N_{A1} + N_{T1} \end{cases}$$

For no asymmetric thrust & thrust line passing thru the C.G.

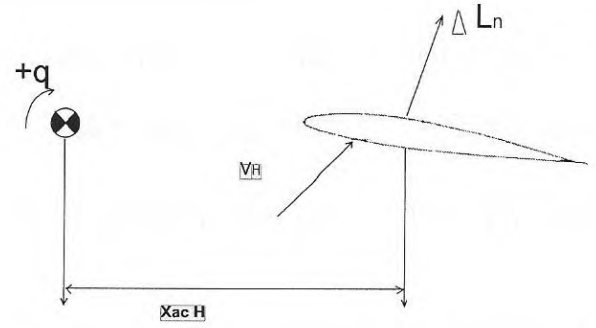
Add aerodynamic forces & moments =>

$$-I_{xz} R_1^2 = M_{A1} =$$

$$(c_{m_0} + c_{m_\alpha}\alpha + c_{m_q}(\frac{Q_1 \bar{c}}{2u_1}) + c_{m_{iH}} i_H + c_{m_{\delta e}} \delta_e).\bar{q} S \bar{c}$$

Term $C_{mq} < 0$ for normal configuration

95-I



C_{mq} : pitch damping derivative or damping in pitch
(Imp. For handling quality)

C_{mq} = f(α , M , Aeroelastic effect) ;

95-II

As a result of an ↑ in pitch velocity a moment must be generated to oppose the ↑ in pitching velocity .

Represents a resistance to rotation in pitch.

$c_{nr} < 0$, Yaw-damping derivative (Damping in yaw) :
as a result of an ↑ in yawing velocity , a moment must be generated to oppose the ↑ in yawing velocity; resistance in yaw.

95-III

Unknowns are β_1 , R_1 , Q_1 , α_1 , δ_{A_1} , i_H , δ_{e_1} , δ_{R_1} and φ_1 , Assuming T and q are known.

R_1 and $Q_1 = f(\psi, \varphi_1)$ => 8 unknowns

Select i_H and φ_1 => 6 equations ; 3 forces and 3 moments;

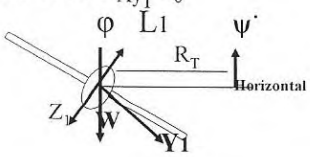
Equations are non-linear and transcendental (coupled) , no implicit solutions.

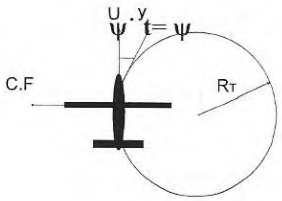
95-IV

Simple method :

Coordinated turn ; $\beta \neq 0$ No y acceleration

Special case $F_{Ay1} = 0$





$$C.F = L\sin\varphi_1 \qquad C.F = W\frac{\sin\varphi_1}{\cos\varphi_1} = mg.\tan\varphi_1$$

$$(C.F = \frac{mu_1^{\,2}}{R}) \quad ; \quad \frac{mu_1^{\,2}}{R_T} = mg.\tan\varphi_1$$

$$mg = L\cos\varphi_1$$

$$\Rightarrow R_T = \frac{u_1^{\,2}}{g.\tan\varphi_1}$$

R_T - radius of turn

96-I

Load factor $n = \dfrac{L}{W}$

$$\Rightarrow n\cancel{W}\cos\varphi_1 = \cancel{W} \Rightarrow n = \frac{1}{\cos\varphi_1}$$

$$C.F = \frac{m(\dot{\psi}_1 R_T)^2}{R_T} = m\dot{\psi}_1^{\,2} R_T$$

$$\Rightarrow mg\tan\varphi_1 = m\dot{\psi}_1^{\,2} R_T$$

$$\Rightarrow \dot{\psi}_1^{\,2} = \frac{g\tan\varphi_1}{R_T} = \frac{g^2\tan^2\varphi_1}{u_1^{\,2}}$$

$$\Rightarrow \dot{\psi}_1 = \frac{g\tan\varphi_1}{u_1}$$ Rate of turn

96-II

Angular Velocity :

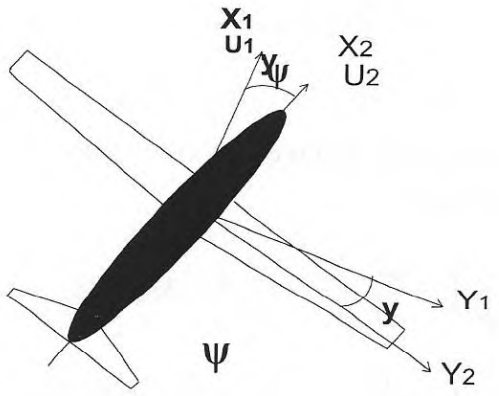
Pitch

$$Q_1 = \dot{\psi_1} \sin\varphi_1$$

$$\Rightarrow Q_1 = (\frac{g\tan\varphi_1}{u_1}).\sin\varphi_1 = \frac{g}{u_1}.(\frac{\sin^2\varphi_1}{\cos\varphi_1})$$

Yaw $\quad R_1 = \dot{\psi_1} \cos\varphi_1$

96-III



96-IV

$$Q_1 = \frac{g(1-\cos^2\varphi_1}{u_1 \quad \cos\varphi_1} \Rightarrow Q_1 = \frac{g}{u_1}(n - \frac{1}{n})$$

$$R_1 = \frac{g}{u_1}\tan\varphi_1 \cos\varphi_1 = \frac{g}{u_1}\sin\varphi_1 = \frac{g}{u_1}.\sqrt{1-\cos^2\varphi_1} \Rightarrow$$

$$R_1 = \frac{g}{nu_1}\sqrt{n^2-1}$$

This means that $F_{Ay_1} = 0$ =>

$$mu_1 R_1 = mg\sin\varphi_1 + \overset{0}{F_{Ay_1}} \Rightarrow R_1 = \frac{g}{u_1}\sin\varphi_1$$

Note that a steady level turn is by definition a coordinated turn (ball- in –center).

97-I

No acceleration component along the Y-axis
(does not mean that $\beta = 0$).
For the full equation , select φ_1 ,
set $F_{AY_1} = 0$ => uncoupled equation
For the lateral system , solve for β_1 ,δ_A and δ_{R_1}

97-II

$F_{Ay_1} = 0$

$$(c_{y\beta}\beta + c_{yr}\frac{R_1 b}{2u_1} + c_{y_{\delta A}}\delta_A + c_{y_{\delta r}}\delta_r)q_1 S = 0$$

Note : this equation is satisfied for $\beta \neq 0$

$$\begin{vmatrix} c_{y_\beta} & c_{y_{\delta a}} & c_{y_{\delta r}} \\ c_{l_\beta} & c_{l_{\delta a}} & c_{l_{\delta r}} \\ c_{n_\beta} & c_{n_{\delta a}} & c_{n_{\delta r}} \end{vmatrix} \begin{vmatrix} \beta \\ \delta a \\ \delta r \end{vmatrix} = \begin{vmatrix} -c_{y_r}\frac{bg.\sin\varphi}{2u_1^2} \\ (I_{zz}-I_{yy})\frac{g^2\sin^3\varphi}{q_1 S b u_1^2\cos\varphi} - c_{l_r}\frac{bg\sin\varphi}{2u_1^2} \\ \frac{I_{xz}g^2\sin^3\varphi}{q_1 S b u_1^2\cos\varphi} - c_{n_r}\frac{bg\sin\varphi}{2u_1^2} \end{vmatrix}$$

Solve for β_1 , δ_A , δ_r as long as the flow is attached. i.e.
δ_A or $\delta_r < 25°$

97-A-II

For longitudinal case , select φ_1 , i_H
and solve for α_1 and δ_{e_1} which indicate the values of α_1 and δ_{e_1} for steady level turn.

97-A-I

For the longitudinal case :

$$\begin{bmatrix} c_{L_\alpha} & c_{L_{\delta e}} \\ c_{m_\alpha} & c_{m_{\delta e}} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \delta_{e_1} \end{bmatrix} = \begin{bmatrix} nc_{L_{trim}} - c_{L_0} - c_{L_{iH}} i_H \\ -c_{m_0} - c_{m_{iH}} i_H - c_{m_q} \dfrac{\bar{c}g}{2u_1^2}(n - \dfrac{1}{n}) \end{bmatrix}$$

Solve for α_1 and δ_{e_1} =>

$$\alpha_1 = \frac{(nc_{L_{trim}} - c_{L_0} - c_{L_{iH}} i_H)c_{m_{\delta e}} + (c_{m_0} + c_{m_{iH}} + c_{m_q} \dfrac{\bar{c}g}{2u_1^2}(n - \dfrac{1}{n}))c_{L_{\delta e}}}{(c_{L_\alpha} c_{m_{\delta e}} - c_{m_\alpha} c_{L_{\delta e}})}$$

$$\delta_e = \frac{-c_{L_\alpha}(c_{m_0} + c_{m_{iH}} i_H + c_{m_q} \dfrac{\bar{c}g}{2u_1^2}(n - \dfrac{1}{n})) - c_{m_\alpha}(nc_{L_{trim}} - c_{L_0} - c_{L_{iH}} i_H)}{(c_{L_\alpha} c_{m_{\delta e}} - c_{m_\alpha} c_{L_{\delta e}})}$$

97-B-I

Note amount of elevator required to trim A/C in a turn =f (n associated with the turn) .

Of practical interest is $\partial\delta_e / \partial n$ " elevator vs. load factor gradient "

Note : the thrust is sufficient to sustain the a/c in its steady state banked flight.

$$\frac{\partial \delta_e}{\partial n} = \frac{-c_{L_\alpha} c_{m_q} \dfrac{\bar{c}g}{2u_1^2}(1 + \dfrac{1}{n^2}) - c_{m_\alpha} c_{L_{trim}}}{c_{L_\alpha} c_{m_{\delta e}} - c_{m_\alpha} c_{L_{\delta e}}}$$

97-B-II

The change in elevator needed to produce a change in load factor

$\partial\delta e / \partial n$ -reflect to as " elevator per g " < 0 for flying quality requirements.

$$\frac{\partial \delta_e}{\partial n} = 0 \Rightarrow -C_{L_\alpha} C_{m_q} \frac{\bar{C}g}{2u_1^2}\left(1+\frac{1}{n^2}\right) - (\bar{x}_{cg} - \bar{x}_{ac}) C_{L_{trim}} = 0$$

$$\Rightarrow \bar{X}_{cg}\Big|_{\frac{\partial \delta_e}{\partial n}=0} = (manuvering\ \ pt) = \bar{X}_{ac} - \frac{C_{m_q}}{C_{L_{trim}}} \frac{\bar{C}g}{2u_1^2}\left(1+\frac{1}{n^2}\right)\ \ n>1$$

As long as $C_{m_q} < 0 \Rightarrow M.p$ is behind the A.C or (stick fix N.P)

Since $C_{L_{trim}} = \frac{w}{\bar{q}S} \Rightarrow$

$$\bar{X}_{c.g}\Big|_{\frac{\partial \delta_e}{\partial n}=0} = M_{P_{fix}} = \bar{X}_{ac} - \frac{C_{m_q}}{4w}\rho S\bar{C}g\left(1+\frac{1}{n^2}\right) = N_{P_{fix}} - \frac{C_{m_q}}{4w}\rho S\bar{C}g\left(1+\frac{1}{n^2}\right)$$

Note : the distance between the M.P and the a.c is a function of h.

At high alt. M.P tends to move toward the a.c.

Now MM : maneuvering margin $MM = MP_{fix} - \bar{X}_{CG}$

An A/C has the correct sign of the " elevator – per-g" gradient

$$\left(\frac{\partial \delta_e}{\partial n} < 0\right), \text{ if } MM > 0$$

If a/c was not turning $\Rightarrow \bar{X}_{cg} = \bar{X}_{ac}$ *ie* $C_{m_q} = 0$

$$\frac{\partial \delta_e}{\partial n} = -\frac{C_{m_\alpha} C_{L_{trim}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}}} \quad , \quad C_L = nC_{L_{trim}} \Rightarrow \frac{\partial C_L}{\partial n} = C_{L_{trim}}$$

$$\Rightarrow \frac{\partial \delta_e}{\partial n} = \frac{\partial \delta_e}{\partial c_L} \cdot \frac{\partial c_L}{\partial n} = c_{L_{trim}} \frac{\partial \delta_e}{\partial c_L} \qquad \text{as before}$$

Turning is a way of reorienting the flight path of an A/C in a horizontal plane .

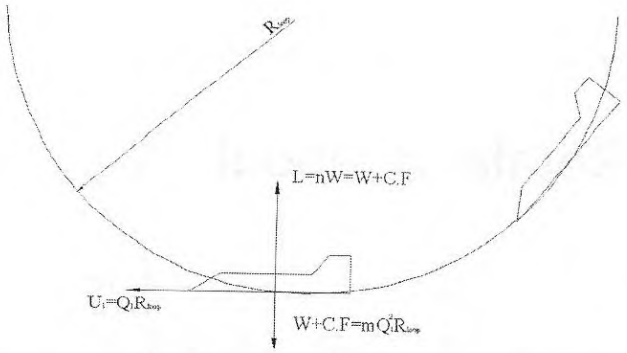
Now reorientation in a vertical plane

- Steady State Symmetric Pull-up

$\dot{\bar{W}} = \dot{\bar{V}} = 0,$ use the longitudinal eq.

The only angular pitch rate is Q_1 : only long. Eq.

only Q_1



$$0 = -(C_{D_0} + C_{D_\alpha}\alpha_1 + C_{D_{i_H}} i_H + C_{D_{\delta_e}} \delta_{e_1})\bar{q}S + T_1 \cos(\phi_T + \alpha_1) \quad (5.113)$$

$$-mU_1 q_1 - mg\cos\gamma_1 =$$

$$-(C_{L_0} + C_{L_\alpha}\alpha_1 + C_{L_q}\frac{Q_1\bar{C}}{2U_1} + C_{L_{i_H}} i_H + C_{L_{\delta_e}}\delta e_1)\bar{q}S - T_1 Sin(\phi_t + \alpha_1)$$

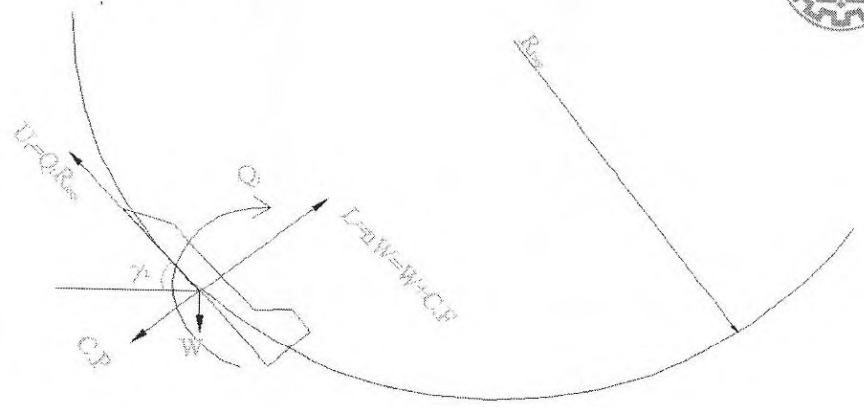
$$0 = (C_{m_0} + C_{m_\alpha}\alpha_1 + C_{m_q}\frac{Q_1\bar{C}}{2U_1} + C_{m_{i_H}} i_{H_1} + C_{m_{\delta_e}}\delta_{e_1})\bar{q}S\bar{c} - T_1 d_T$$

Unknowns are $\alpha_1, i_{H_1}, \delta_{e_1}, Q_1, T_1, \gamma_1, q_1$

(Select $\bar{q}_1, T_1$ and i_{H_1}) Can solve for α_1, Q_1 *and* δ_{e_1}

98-3

$$C.F = mQ_1^2 R_L$$



$$L = mQ_1^2 R_L + mg\cos\gamma_1$$

99

At the bottom of the loop $\gamma = 0 \Rightarrow$

$L = mQ_1^2 R_L + mg$ but L= n w, $u_1 = Q_1 \tilde{R}_L$ (Q_1 : Angular velocity)

from the laws of mechanics

$$nw = mQ_1^2 R_L + mg = mu_1 Q_1 + mg \Rightarrow Q_1 = \frac{g}{u_1}(n-1) \quad for \quad \gamma_1 = 0 \quad \text{only}$$

Now substitute for Q_1 into the 3 longitudinal Eq, can solve for α_1 and δ_{e_1}

99-1

$$\begin{bmatrix} C_{L_\alpha} & C_{L_{\delta_e}} \\ C_{m_\alpha} & C_{m_{\delta_e}} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \delta_{e_1} \end{bmatrix} = \begin{Bmatrix} nC_{L_{trim}} - C_{L_0} - C_{L_{i_H}} i_{H_1} \\ -C_{m_0} - C_{m_{i_{H_1}}} - C_{m_q} \frac{\bar{C}g}{2U_1^2}(n-1) \end{Bmatrix}$$

$$\alpha_1 = \frac{(nC_{L_{trim}} - C_{L_0} - C_{L_{i_H}} i_{H_1}) C_{m_{\delta_e}} + \left[C_{m_0} + C_{m_{i_H}} i_{H_1} + C_{m_q} \frac{\bar{C}g}{2U_1^2}(n-1) \right] C_{L_{\delta_e}}}{(C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}})}$$

$$\delta_{e_1} = \frac{-C_{L_\alpha} \left\{ C_{m_0} + C_{m_{i_H}} i_H + C_{m_q} \frac{\bar{C}g}{2U_1^2}(n-1) \right\} - C_{m_\alpha} \left(nC_{L_{trim}} - C_{L_0} - C_{L_{i_H}} i_H \right)}{\left(C_{L_\alpha} C_{m_{\delta_e}} - C_{m_\alpha} C_{L_{\delta_e}} \right)}$$

These are α_1 and δ_{e_1} required to maintain the A/C in a symmetric pull up .

Now the change in δ_e req. is a result of a change in load factor.

then

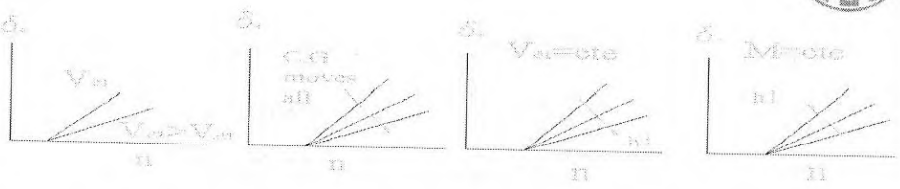
$$\frac{\partial \delta_e}{\partial n} = \frac{-C_{L_\alpha} C_{m_q} \frac{\bar{c}g}{2U_1^2} - C_{m_\alpha} C_{L_{trim}}}{C_{L_\alpha} C_{m_{\delta_e}} - C_{L_{\delta_e}} C_{m_\alpha}},$$

$$\frac{\partial \delta_e}{\partial n} = 0 \Rightarrow$$

$$M.P\Big|_{pull\ up} = \bar{X}_{CG}\Big|_{\frac{\partial \delta_e}{\partial n}=0} = \bar{X}_{ac} - \frac{C_{m_q}}{C_{L_{trim}}} \frac{\bar{C}g}{2U_1^2}$$

$$C_{L_{trim}} = \frac{2w}{\rho S U_1^2} \Rightarrow \bar{X}_{CG} = \bar{X}_{ac} - \frac{C_{m_q}}{4w} \rho S \bar{C} g$$

$\frac{2W}{\rho \bar{C} T}$ – *relative mass parameter* or $M_{P_{fix}} = N_{P_{fix}} - \frac{C_{m_q} \cdot \rho S \bar{C} g}{4w}$



Note : $M_{P_{fix}}$ in steady turn is always fwd. of $M_{P_{fix}}$ in a symm. Pull up .

Note: for both cases, steady turn and symm. Pull up ,

$$M_{P_{fix}} \rightarrow N_{P_{fix}} \rightarrow N_{P_{fix}} \quad \text{at very high h}$$

99-3

For A/C with tail $C_{m_q} = -2Ka_t \frac{L_t}{c}\bar{V}_H$

K= Const. ⊔ 1.1 for straight winged A/C

$$\Rightarrow \bar{X}_{C.G} = \bar{X}_{ac} + \frac{Ka_t}{W}\rho S g L_t \bar{V}_H$$

- At high alt., the maneuver point approaches the a.c.
- Wing loading $\frac{w}{S}$ has great influence on maneuvering point.
- Tail size , L_t has great influence on maneuvering point.

100

192

چطور یک خلبان به هدایی توانایی کنترل هواپیما را دارد؟

Control systems

How well a pilot is able to control the A/C?

یک راه که خلبان اساس پایداری طولی استاتیکی در هواپیما کند نیروی مورد نیاز stick برای جبران فعال لولای elevator

A way in which the pilot " feels" the static longitudinal stability of the A/C is the stick force req. of the moment acting of the hinge of the elevator.

این جبران hinge یعنی که خلبان باید به وسیله اعمال کردن یک نیرو به control stick تلاش کند.

This hinge moment is the moment that pilot must overcome by exerting a force on the control stick

Two classes of control systems:

۱- سیستم کنترل پرواز برگشت پذیر که حرکت از نتیجه کنترل از جایی یک پیوند مکانیکی مستقیم یعنی کابلها - میله فشاری و ... یک نتیجه حرکت سیستم کنترل در cockpit حرکت شرایط و برعکس (در جهت مخالف)

1. Reversible flight control system: where movement of the cockpit controls result in movement of the surface control through a direct mechanical linkage i.e. cables, push – rods , etc. as a result movement of the control systems result in movement of the cockpit condition and vice versa .

100-1

۲- سیستم کنترل پرواز برگشت ناپذیر حرکت کنترلهای cockpit نتیجه می دهد در حرکت کنترل سطوح آیرودینامیکی که هم جزئی یا کلی با کمک گرفتن از بکارگیری دستگاه الکتروهیدرولیک یا الکترومکانیک. حرکت کنترلهای سطوح آیرودینامیکی (با فعالیت دستگاه قدرت) معمولاً هرگز ممکن نیست. اگر هر تغییراتی در فشار آیرودینامیکی فاصله بالای سطوح کنترل، بازگشتی به کنترلهای cockpit ندارد.

2. Irreversible flight control system: movement of the cockpit controls result in movement of the aerodynamic surface control either partially or completely with the help of an electro-hydraulic and /or electro mechanical actuation device. Movement of the aerodynamic surface controls (with the actuation device powered up) is usually not even possible. If any changes in aerodynamic pressure dist. over control surface no feedback to cockpit controls.

۳- سیستم پیوندی هردو سیستمها انتقال می دهند ورودی خلبان را به پدنهٔ هواپیما

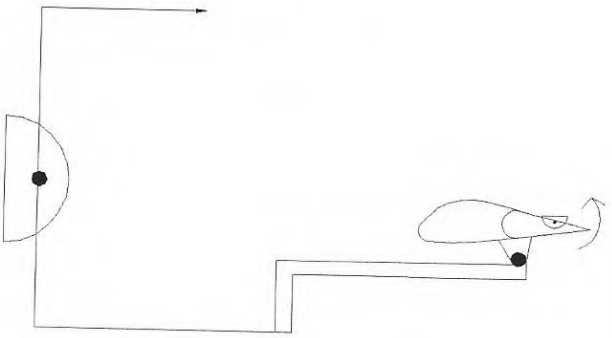
3 Hybrid system both systems transmit pilot input to the airframe.

101

-Pull force (defined as positive) should always lift the nose & slow the A/C

-Push force (defined as negative) should always drop the nose & speed up the A/C.

$F_S = G.HM$



G : Gear ratio measure of mechanical load provided by the control system

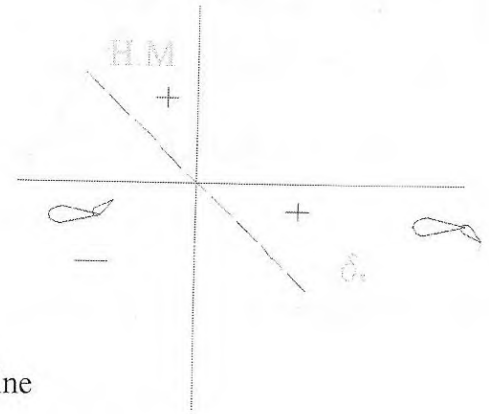
101-1

H.M - Hinge moment created by aerodynamic forces

$$HM = \bar{q}_H S_E C_E C_h \Rightarrow F_S = G\bar{q}_H S_E C_E C_h$$

$F_S \uparrow$ with A/C sizing & V^2

same can be obtained for rudder & aileron



S_E – area aft of the hinge line

101-2

$$C_h = C_{h_0} + C_{h_{\alpha_H}} \alpha_H + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \delta_t + C_{h_{i_H}} i_H$$

$$C_{h_{\alpha_H}} = \frac{dC_h}{d\alpha_H}\ ,\ C_{h_{\delta_e}} = \frac{dC_h}{d\delta_e}\ ,\ C_{h_{\delta_t}} = \frac{dC_h}{d\delta_t} : \delta_t - tab$$

$C_{h_0} = 0$ For symm. tail

Hinge moments are very difficult to predict analytically, must use wind tunnel tests.

101-3

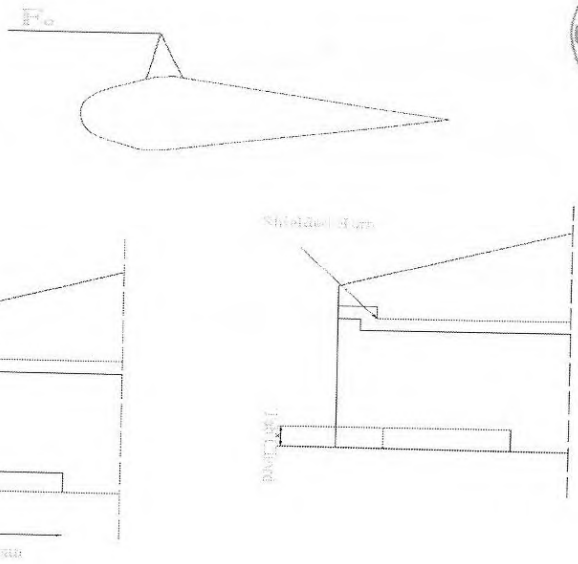
Experience indicated that even with the best of predication methods hinge moment can often be predicted only to within ± 30% accuracy.

This is because C_p dist. over the aft portion of control surface is strongly affected by the B.L $\Rightarrow$ Re is very important.

The following parameters affect the value of hinge moment coefficient.

- Re
- Control surface deflection
- Nose shape
- Horn geometry
- α
- T.E angle
- Mach No.
- Chord ratio
- Gap
- Tab geometry
- Overhang

101-A



101-A-1

Basically 4 types of hinge moment to be considered.

-Hinge moment for $\alpha_h = \delta_e = \delta_{t_e} = 0$ (for symmetric airfoil=0)

-Hinge moment for $\alpha_h \neq 0$ $\delta_e = \delta_{t_e} = 0$

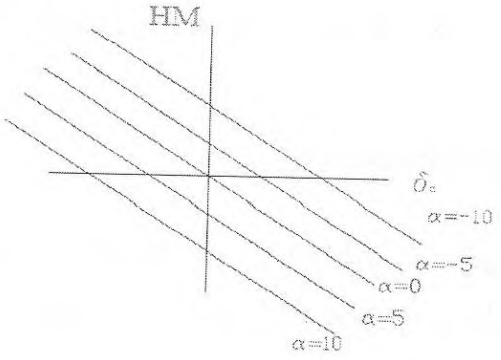
-Hinge moment for $\alpha_h = 0$, $\delta_e \neq 0$, $\delta_{t_e} = 0$

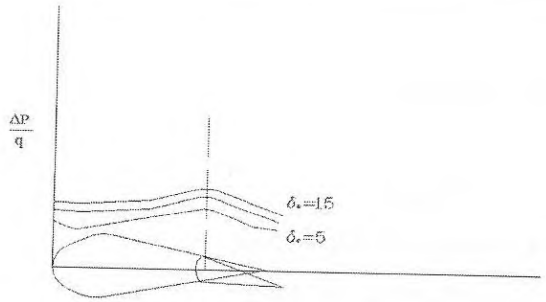
-Hinge moment for $\alpha_h = 0$, $\delta_e = 0$, $\delta_{t_e} \neq 0$

101-A-2

Magnitude of hinge moment

For any α and control surface deflection is developed from the complicated pressure distribution





$$\alpha_H = \alpha - \varepsilon + i_H = \alpha\left(1 - \frac{d\varepsilon}{d\alpha}\right) + i_H - \varepsilon_0$$

$$F_S = G\bar{q}_H S_e \bar{C}_e \left[C_{h_0} + C_{h_\alpha}\left(\left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha + i_H - \varepsilon_0\right) + C_{h_{\delta_e}}\delta_e \right]$$

Stick force for zero tab deflection.

For Gliding flight:

$$\alpha_{trim} = \alpha_{0_{trim}} + \frac{\partial \alpha}{\partial C_L} C_{L_{trim}} \qquad 5.44$$

$$\delta_{e_{trim}} = \delta_{e_{0_{trim}}} + \frac{\partial C_{m_{cg}}}{\partial C_L} C_{L_{trim}} \qquad 5.45$$

$\alpha_{0_{trim}}$ & $\delta_{e_{0_{trim}}}$ = const with respect to C.G

Substitute for $F_S \Rightarrow$

$$F_S = \bar{q}_H S_E C_E G \left\{ C_{h_0} + C_{h_\alpha} (1 - \frac{d\varepsilon}{d\alpha})(\alpha_{0_{trim}} + \frac{\partial \alpha}{\partial C_L} \frac{w}{\bar{q}S}) + C_{h_\alpha} (i_H - \varepsilon_0) + C_{h_{\delta_e}} \right.$$

$$\left. (\delta_{0_{trim}} + \frac{\partial \delta_e}{\partial C_L} \frac{w}{\bar{q}S}) \right\} \qquad 5.136$$

Stick force to trim the A/C:

102-2

Max. Allowable stick forces are fixed in the regulation. (All allowable cockpit control forces in terms of temporary & prolonged flight cond.

- ***Temporary condition***
  - Rudder pedal forces requirement following a sudden outboard engine failure.
  - Elevator stick forces requirement following a sudden outboard engine failure.

102A.I

شرایط پرواز طولانی:
ظاهر می‌شود در شرایط پروازی
مادی یا بعداز اینکه هواپیما
دوباره trim شد به وسیله
خلبان بعداز تعدادی خرابی.

. *Prolonged flight condition*

Occurs in normal flight condition or after the A/C has been re-trimmed by the pilot following some failure.

Max. cockpit control forces allowed by far 23, 25 (lbs)

. *Temp. Application*

	Pitch	Roll	yaw
Stick	60	30	
Wheel (applied to trim)	75	60	
Rudder pedal			150
. Prolonged App.	10	5	20

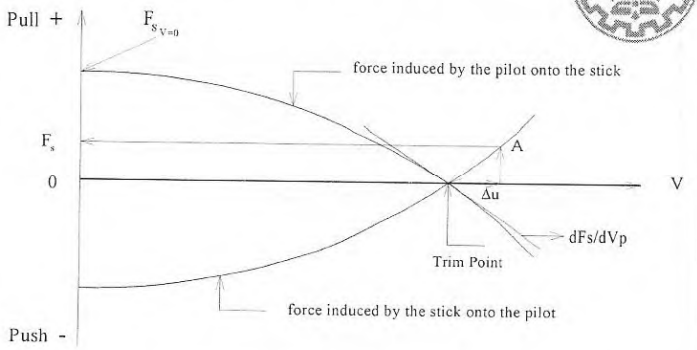
102A.II

$$\therefore \quad F_S = F_{S_{V_P=0}} + \frac{1}{2}\frac{\partial F_S}{\partial V_P} V_p$$

$F_{S_{V_p=0}}$: Stick force mag. at zero speed

Use $\eta_H = \frac{\bar{q}_H}{q}$ and take $\frac{\partial F_S}{\partial V_p} \Rightarrow$

$\frac{\partial F_S}{\partial V_P}$: Stick force speed gradient & is very important to pilots.



If A/C is perturbed by horizontal gust of mag. Δu and the corresponding pull-force or the stick, F_s, the A/C is returned back to it's trimmed, $u_{trimmed}$

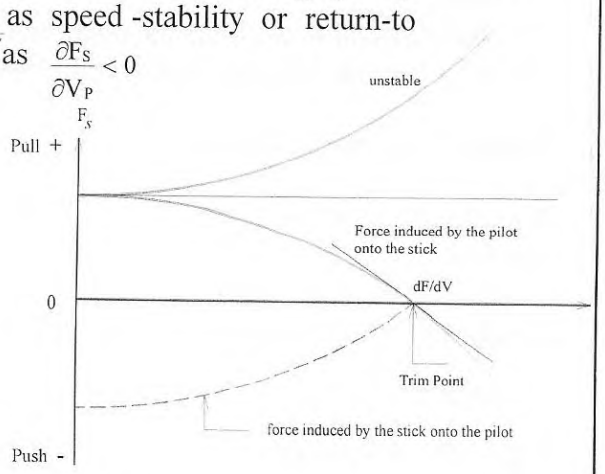
A typical min. allowable slope is 1 lbs per 6 Knts of speed.

102B.I

$$\frac{\partial F_S}{\partial V_P} = \rho \eta_H V_P S_e c_e G \left[c_{h_0} + c_{h_\alpha}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_{0_{trim}} + c_{h_\alpha}\left(i_H - \varepsilon_0\right) + c_{h_{\delta_e}}\delta_{e_{0_{trim}}} \right]$$

Stick force gradient < 0 for correct handling quality & the a/c referred to as speed -stability or return-to -trim -stability as long as $\frac{\partial F_S}{\partial V_P} < 0$

Stick force trim
$Fs = 0$
Use i_H to zero out the stick force



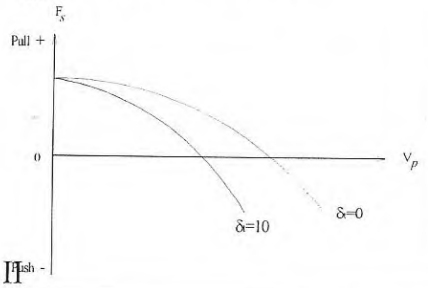
103.I

- If A/C slows down, a negative stick force occurs which rotate the nose of the A/C downward ⟶ ↑ velocity.

- If A/C speed exceeds V_{trim} , a positive (pull) stick force causes the A/C nose to pitch up ⟶ ↓ velocity.

Also as C.G. is moved aft. , $\frac{\partial F_S}{\partial V_P}$, becomes lower.

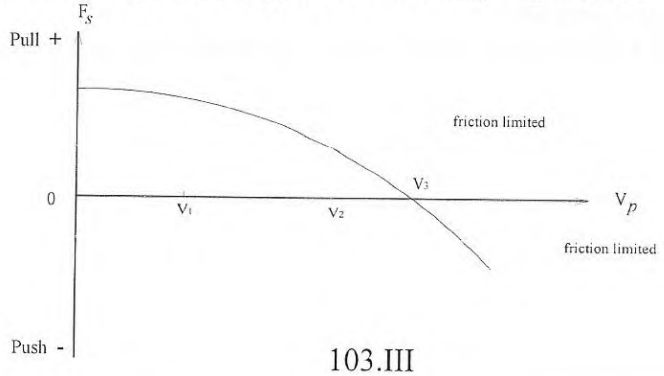
To calculate i_H for the case when $F_S = 0$. set , $F_S = 0$ solve for i_H , Eq. 5.140



103.II

- It is important to keep friction in longitudinal control system very low so as not to mask the feel completely.

- If the A/C with it's tab set for trim at V_3 is slowed down to V_1 , upon release of the stick , the speed would ↑ up to V_2 .The force gradient inside the friction left is masked & gives an apparent instability < 1 or 2 lbs.



103.III

$\frac{dF_S}{dV_P}$ is extremely important, it plays major role in determining the pilot's feel of the A/C. (pilot concept of stability).

Large gradient will:

-Keep the A/C flying at constant speed.
-Resist the influence of disturbance toward changing the speed.
-Enable the pilot to trim the A/C out easily.
-Not require a lot of pilot attention to ~~hold~~ the given trim speed.

103A.I

. Effect of deflecting a trim tab can be thought of as a change in the hinge moment coefficient c_{h_0} , if

$$c_{h_0} \rightarrow c_{h_0} + c_{h_{\delta_t}} \delta_t = c_{h_{0_{trim}}}$$

Solve Eq. for $F_S = 0$ with values of $c_{h_{0_{trim}}}$

$$c_{h_{0_{trim}}} = -\left\{ c_{h_\alpha} \left(1 - \frac{d\varepsilon}{d\alpha} \right) \alpha_{C_{L_1}=0} + c_{h_\alpha} (i_h - \varepsilon) + c_{h_{\delta_e}} \delta_{e_{C_{L_1}=0}} \right\}$$

$$- c_{L_1} \left\{ \frac{c_{h_\alpha} c_{m_{\delta_e}} \left(1 - \frac{d\varepsilon}{d\alpha} \right) - c_{h_{\delta_e}} c_{m_\alpha}}{c_{L_\alpha} c_{m_{\delta_e}} - c_{m_\alpha} c_{L_{\delta_e}}} \right\}$$

c_{h_0} can be altered by changing the gearing ratio on a geared tab. Easy during flight testing since other parameter are not easy to change.

103A.II

Stick Free:

Elevator is allowed to float usually associated with A/C that have reversible control system.

. The control surface "float" is a function of the moment generated by the elevator about it's hinge line.

. The pilot trims the A/C by adjusting the tab and the elevator at a certain $C_L \Rightarrow H.M. = 0$

Now when the a/c is subject to a small disturbance & the elevator is left free to float. In this condition , the elevator takes up an angle determined by the condition that H.M.=0

103B.I

Hinge moment is caused by 2 factories:

- Tendency of the elevator to streamline itself with the relative wind variation of the elevator H.M. with H.T. A.O.A.
- 2^{nd} factor affecting the "float" is the H.M. generated by the elevator deflection when the H.T. is at zero A.O.A.

When the elevator is in equilibrium. $\Rightarrow H.M. = 0$,the floating tendency $c_{h_{\alpha_t}}$ is cancelled by the "restoring tendency".

103B.II

Stick free: "elevator free"

One of the most characteristic of any control surface is it's so called floating characteristic. H.M. = 0

$$c_h = c_{h_0} + c_{h_\alpha}\alpha_H + c_{h_{\delta_e}}\delta e = 0$$

$\therefore$ The floating angle is $\delta_{e\,free} = -\dfrac{c_{h0} + c_{h_\alpha}\alpha_H}{c_{h_{\delta_e}}}$

$\therefore$ δ_e depends on c_{h_α}(floating tendency) and $c_{h_{\delta_e}}$ (restoring tendency); but as soon as the control surface starts to float up , a new pressure dist. due to $c_{h_{\delta_e}}$ is added to prevent the elevator from floating up.

104.I

$$\delta_{e\,free} = -\frac{c_{h0}}{c_{h_{\delta_e}}} - \frac{c_{h_\alpha}}{c_{h_{\delta_e}}}\left[\alpha - \left(\varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha\right) + i_H\right]$$

$$\delta_{e\,free} = \delta_{e_{0\,free}} + \frac{\partial\delta_e}{\partial\alpha}\alpha \qquad \text{but}$$

$$\therefore \delta_{e_{0\,free}} = -\frac{c_{h0}}{c_{h_{\delta_e}}} - \frac{c_{h_\alpha}}{c_{h_{\delta_e}}}(i_H - \varepsilon_0)$$

$$\frac{\partial\delta_e}{\partial\alpha} = -\frac{c_{h_\alpha}}{c_{h_{\delta_e}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad , c_{h_\alpha} < 0 \quad c_{h_{\delta_e}} < 0 \ ,$$

$\therefore$ The elevator will float up more & more as $\alpha\uparrow$; It's floating characteristic depends on $c_{h_{\delta_e}}$ & c_{h_α} .

104.II

Want to reduce the floating tendency as near zero as practicable i.e. c_{h_α} should be made to have a zero slope.

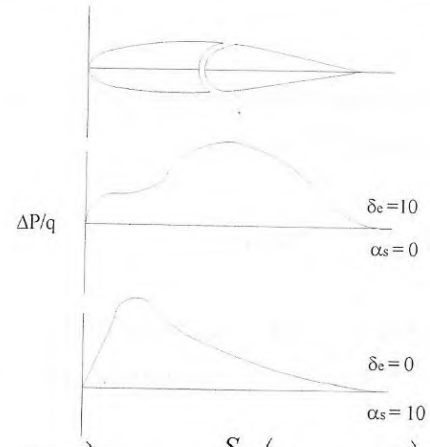
$c_{h_{\delta_e}}$ must be made to have a low slope so that the pilot can move the control especially for large , fast A/C.

Controlling C_{h_α} & $C_{h_{\delta_e}}$ are important & methods are referred to "aerodynamic balancing control surface "

One method is to move the hinge line aft. ⟶ the total moment will reduce.

If move the hinge line way back , the resultant moment will reverse ⟶ the control is said to be overbalanced.

104A.I



Remember c_{m_α}

$$c_{m_\alpha}\Big|_{stick-free} = c_{L_{\alpha_{WB}}}\left(\bar{X}_{cg} - \bar{X}_{ac_{WB}}\right) - c_{L_{\alpha_H}}\eta_H \frac{S_H}{S}\left(\bar{X}_{ac_H} - \bar{X}_{cg}\right)\left\{\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \tau_e \frac{\partial \delta_e}{\partial \alpha}\right\}$$

$$\tau_e = \frac{\partial \alpha_H}{\partial \delta_e}$$

104A.II

Substitute for $\frac{\partial \delta_e}{\partial \alpha}$ and set $C_{m_\alpha}=0$ $\qquad \bar{X}_{cg} = \bar{X}'_{ac}$

$$\bar{X}_{cg}\Big|_{C_{m\alpha}=0} = NP_{free} = \frac{\bar{X}_{ac_{wb}} + \frac{C_{L_{\alpha H}}}{C_{L_{\alpha WB}}}\eta_H \frac{S_H}{S}\bar{X}_{acH}\left(1-\frac{d\varepsilon}{d\alpha}\right)\left(1-\frac{C_{h\alpha}\tau_e}{C_{h\delta_e}}\right)}{1+\frac{C_{L\alpha H}}{C_{L\alpha WB}}\eta_H \frac{S_H}{S}\left(1-\frac{d\varepsilon}{d\alpha}\right)\left(1-\frac{C_{h\alpha}\tau_e}{C_{h\delta_e}}\right)}$$

the denominator is usually ≈ 1

$$\rightarrow \quad NP_{free} \approx \bar{X}_{ac_{WF}} + \left[1-C_{h\alpha_t}\left(1-\frac{C_{L_{\delta_e}}}{C_{L\alpha_t}}\frac{C_{h\alpha_t}}{C_{h_{\delta_e}}}\right)\right] V_H \eta_H \frac{C_{L_{\alpha H}}}{C_{L_{\alpha W}}}\left(1-\frac{d\varepsilon}{d\alpha}\right)$$

[ ] term determines whether stick free N.P lies fwd. or aft. of stick fixed N.P

104B-I

$$\rightarrow \quad NP_{free} \cong NP_{fix} + \frac{C_{L\alpha_H}}{C_{L\alpha_{WB}}}\eta_H \frac{S_H}{S}\bar{X}_{ac_h}\left(1-\frac{d\varepsilon}{d\alpha}\right)\left(\frac{C_{h_\alpha}\tau_e}{C_{h_{\delta_e}}}\right)$$

$$\cong NP_{fix} - \frac{C_{m_{\delta_e}}}{C_{L_\alpha}}\left(1-\frac{d\varepsilon}{d\alpha}\right)\left(\frac{C_{h_\alpha}}{C_{h_{\delta_e}}}\right) \qquad (*)$$

usually N.P stick-free is fwd. of N.P stick fixed

Stick-free maneuver stability is higher than the stick-free stability in level flight & the stick-free maneuver pt. is aft. of the stick-free neutral pt.

104B-II

To justify eq^{n} (*) , remember that:

$$C_{m\delta_e} \cong -C_{L\alpha_H} \frac{S_H}{S} \eta_H \tau_e \left(\bar{X}_{ac_h} - \bar{X}_{cg}\right)$$

Now

Def. 1:

Static margin stick fixed = S M_{fix} $= NP_{fix} - \bar{X}_{cg} = \bar{X}_{acA} - \bar{X}_{cg}$

Def. 2:

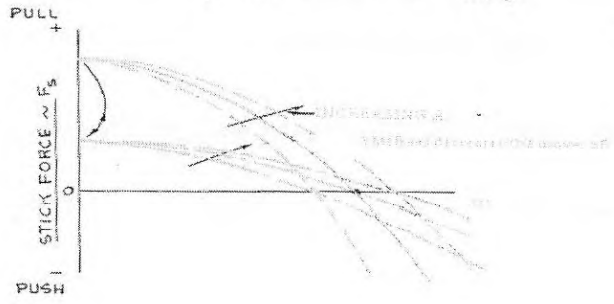
Static margin stick free = SM_{free} = $NP_{free} - \bar{X}_{cg}$

Combining the above two def. ➔

105A-I

$SM_{fix} - SM_{free} = NP_{fix} - NP_{free}$

$$= \left\{ \frac{C_{m\delta_e}}{C_{L\alpha}} \left(1 - \frac{d\varepsilon}{d\alpha}\right) \frac{C_{h\alpha}}{C_{h\delta_e}} \right\}$$



105-A-II

Usually $\overline{X}'_{ac} \prec \overline{X}_{ac}$, i.e stick free N.P is generally fwd of the A/C aerodynamic center NP_{fix}.

After some algebra:

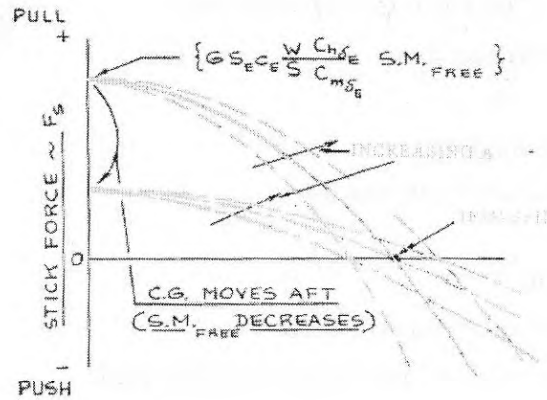
$$F_s = \eta_H G S_E C_E \left\{ \bar{q} A + \frac{W}{S} \frac{C_{h_\delta}}{C_{m_\delta}} S.M_{free} \right\} \qquad (**)$$

$$A = C_{h0} + C_{h_\alpha} \left[(i_h - \varepsilon_0) + \left(1 - \frac{d\varepsilon}{d\alpha} \right) \alpha_{0trim} \right] + \delta_{0trim} C_{h_\delta}$$

$$\frac{\partial F_s}{\partial V_p} = \rho \eta_H G S_E C_E A U_1$$

to ↑A, we can ↓i_h or ↓δ_T (tab deflection)

105-I



105-II

Stick force F_s

1-Directly proportional to G(gearing ratio)

2-Proportional to $S_E C_E$, the cube of A/C size.rapid↑ in F_s with size

3-C.G only affects the const term (apart from a second-order influence on $C_{m\delta}$) fwd. movement of C.G produces an upward translation of the curve .

4-Weight of the A/C thru W/S .When W/S↑ ,translates the curve upward.

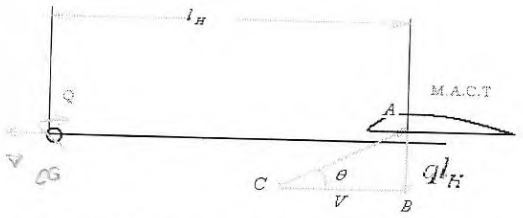
5-F_s varies with $\frac{1}{2}\rho V^2$↓with height, and ↑ as the speed squared.

6- The effect of trim tab is to change the coeff. of $\frac{1}{2}\rho V^2$,hence the curvature of parabola. It changes the intersect of the curve with V axis.Vtrim– speed for zero stick force.

Pull-up for const. g(stick force per-g)

Is given the symbol $\frac{\partial F_s}{\partial n}$: a way a pilot can feel the relationship between stick force &severity of maneuver. The effect of aerodynamic rotation on the elevator hinge mom. is taken into account:

$$F_s = \bar{q}_H S_E C_E G\left[C_{h0} + C_{h\alpha}\alpha + C_{h\delta_e}\delta_e\right]$$



AB-velocity vector of tail due to rotation Q

BC-velocity vector of tail due to translation V

θ: change in α_t due to Q

$$\theta = \tan^{-1}\frac{Ql_H}{V} \cong \frac{Ql_H}{V}$$

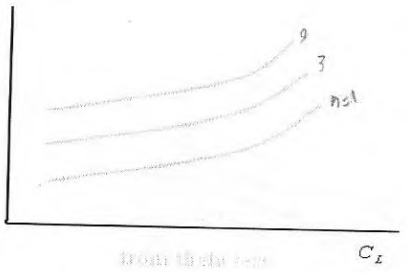
106-III

$$\alpha_H = \alpha - \varepsilon + i_H + \frac{Ql_H}{U_1}$$

$$l_H = \overline{X}_{acH} - \overline{X}_{cg}$$

$$QU_1 = g(n-1)$$

Q: angular vel.



$$\alpha_H = \alpha - \varepsilon + i_H + \frac{gL_h}{U_1^2}(n-1)$$

due to rotation $C_{L_1} \rightarrow nC_{L_1}$ or $C_L = \dfrac{2n\dfrac{W}{S}}{\rho V^2}$

$$nC_{L_1} = C_{L\alpha}\alpha - \alpha_{0L}C_{L\alpha}$$

$$\rightarrow \quad \alpha = \frac{nC_{L_1} + \alpha_{0L}C_{L\alpha}}{C_{L\alpha}} = \alpha_{0L} + n\frac{C_{L1}}{C_{L\alpha}}$$

107-I

$$\alpha_H = \alpha_{0L}\left(1-\frac{d\varepsilon}{d\alpha}\right)-\varepsilon_0+\left(1-\frac{d\varepsilon}{d\alpha}\right)n\frac{C_{L_1}}{C_{L\alpha}}+i_H+\frac{gl_H}{U_1^2}(n-1)$$

as before n=1 ➔ $\delta_e = \delta_{e0_{trim}} + \frac{\partial \delta_e}{\partial C_L} C_{Ltrim}$

For $n \neq 1$ ➔ $\delta_e = \delta_{e0trim} + \frac{\partial \delta_e}{\partial C_L} nC_{L1} - \frac{Ql_H}{U_1}\frac{1}{\tau_e}$

$$= \delta_{e0} - \frac{C_{mq}(n-1)\frac{\bar{C}g}{2U_1^2}}{C_{m_{\dot{\alpha}e}}} - \frac{C_{m\alpha} nC_{L_{trim}}}{C_{L\alpha} C_{m\delta_e}} \quad ; \tau_e = \frac{\partial \alpha_H}{\partial \delta_e}$$

subst. for C_{mq} & $C_{m\dot{\alpha}e}$ ➔

$$\delta_e = \delta_{e0} - (n-1)\frac{1.1g\left(\bar{X}_{ac_h} - \bar{X}_{cg}\right)}{\tau_e U_1^2} + n\frac{C_{Ltrim}}{C_{m\delta_e}} SM_{fix}$$

107-II

and $\frac{\partial \delta_e}{\partial C_L} = \frac{SM}{C_{m\delta_e}} = \frac{\bar{X}_{ac} - \bar{X}_{cg}}{C_{m\delta_e}}$ for $\eta_H \prec 1$

$\delta_{e0} = \delta_{e(n=1 \quad \& C_{Ltrim}=0)}$

$$F_{S_{|PullUp}} = \eta_H \bar{q} S_E C_E \left[A + nC_{L_1} \frac{C_{h\delta_e}}{C_{m\delta_e}} SM_{free} + \frac{n-1}{U_1^2} gl_H \left(C_{h\alpha} - \frac{1.1C_{h\delta_e}}{\tau_e} \right) \right]$$

$$\frac{\partial F_S}{\partial n} = \eta_H \bar{q} S_E C_E \left[C_{L_1} \frac{C_{h\delta_e}}{C_{m\delta_e}} SM_{free} + \frac{gl_H}{U_1^2} \left(C_{h\alpha} - \frac{1.1C_{h\delta_e}}{\tau_e} \right) \right]$$

Set $\frac{\partial F_s}{\partial n} = 0$ and solve for $\bar{X}_{cg} = MP$ for pull-up free

107-III

$\frac{\partial F_S}{\partial n} \succ 0$ as required ,

if $\left[C_{L_1} \frac{C_{h\delta_e}}{C_{m\delta_e}} SM_{free} + \frac{g l_H}{U_1^2} \left(C_{h\alpha} - \frac{1.1 C_{h\delta_e}}{\tau_e} \right) \right] \succ 0$

For an A/C with a reversible flight control system, the maneuver point stick free , MP_{free} is defined as:

"C.G. location where the stick-force-per-g gradient ,vanishes"

$$\frac{\partial F_S}{\partial n} = 0 \Rightarrow$$

$$MP_{free} = \bar{X}_{cg\left|\frac{\partial F_S}{\partial n}=0\right.} = NP_{free} + \left[\frac{g\left(X_{ach} - X_{cg}\right)}{C_{L_1} U_1^2} \right] \left[C_{h\alpha} - \frac{1.1 C_{h\delta_e}}{\tau_e} \right] \frac{C_{m\delta_e}}{C_{h\delta_e}}$$

or

$$MP_{free} = \bar{X}_{cg\left|\frac{\partial F_S}{\partial n}=0\right.} = NP_{free} - \left[1 - \frac{C_{h\alpha} \tau_e}{1.1 C_{h\delta_e}} \right] \frac{\rho S \bar{C} g}{4W} C_{mq}$$

From before

$$MP_{fix} = \bar{X}_{cg}\Big|_{\frac{\partial \delta_e}{\partial n}=0} = NP_{fix} - \frac{C_{mq}}{4W}\rho S\bar{C}g$$

Comparing ➔ the diff. is in $\left[1 - \frac{C_{h\alpha}\tau_e}{1.1C_{h\delta_e}}\right]$ term ➔ elevator hinge moment derivative

For usual negative sign of the hinge moment derivative
➔ MP_{free} is fwd. of MP_{fix}

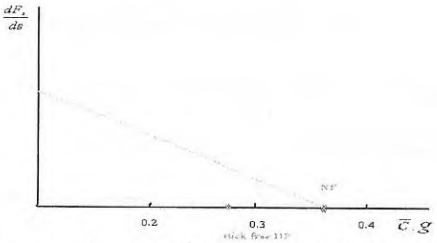
C.G. location is very imp.

∴ Maneuver Margin is reduced with the control free vis-à-vis control fixed

107-A-II

Note that $\frac{\partial F_s}{\partial n}$ do not vanishes at the stick free N.P,
i.e $S.M_{free}=0$

But if C.G. is moved sufficiently aft. of stick free N.P,a position will be reached s.t. $\frac{\partial F_s}{\partial n} = 0$,this C.G. limit is termed stick-free maneuver point N'_m



The stick free M.P is almost always aft. of the stick free neutral pt
-variation of $\frac{\partial F_s}{\partial n}$ with C.G. shift is a function of $C_{h\delta_e}$ only

107-B-I

with the above fig. $\dfrac{d(\frac{F_s}{n})}{d(\%C.G.shift)} = \dfrac{\frac{W}{S} G S_e C_e C_{h\delta}}{100\, C_{m\delta}}$

Very imp. for flight test engineer, by determining this variation ,can extract $C_{h\delta}$

-Also change in stick force/g with C.G. varies with A/C size and wing loading.

An study of damping term $\eta_H G \bar{q} S_e C_e \dfrac{g l_H}{U_1^2}\left(C_{h\alpha} - \dfrac{C_{h\delta_e}}{\tau_e}\right)$

-Stick force in accelerated flight is a function of alt. , fighter pilot report a decay in stability with h.

107-B-II

Note that :

- Stick force /g increases linearly from zero as the C.G. is moved forward from the stick free N.P. and reverses sign if MP does so $_{free}$
- It is related to W/S , high wing loading produces heavier control
- For similar A/C of different size but equal wing loading $\dfrac{\partial F}{\partial n} \propto S_e C_e$, i.e. to the cube of the linear size
- indep. of speed , but the effect of M & Re is important

(108_I)

Trim tabs

- To make Fs=0 (used in reversible flight control systems for getting mom. Trim)

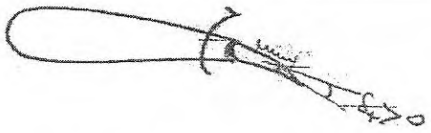
$$F_s = \bar{q} S_e C_e G \left[C_{h_0} + C_{h_{\alpha_h}} \alpha_h + C_{h_{\delta e}} \delta e \right]$$

$$\alpha_h = \alpha - \varepsilon + i_h$$

For n=1 , steady level flight , can trim the A/C by

- Set i_H so that $F_s = 0$, all moveable tail
- For a fixed H-T A/C , most A/C trim by an elevator tab

(108_II)



Deflect the tab by a so called trim-wheel

Rotate until $F_s = 0$

* $C_{h_0} = C_{h_{0_{old}}} + C_{h_{\delta_t}} \delta t$

$$F_s = \eta_H G S_e C_e \left\{ \bar{q} \left(A + C_{h_t} \right) + \frac{W}{S} \frac{C_{h_{\delta e}}}{C_{m_{\alpha e}}} SM_{free} \right\} = 0 \quad (*)$$

$$\Rightarrow C_{h_{trim}} = -A - \frac{1}{q_{trim}} \frac{W}{S} \frac{C_{h_{\delta e}}}{C_{m_{\delta e}}} SM_{free}$$

And $A = C_{h_0} + C_{h_\alpha} \left[\left(i_h - \varepsilon_0 \right) + \dots \right]$

(109_I)

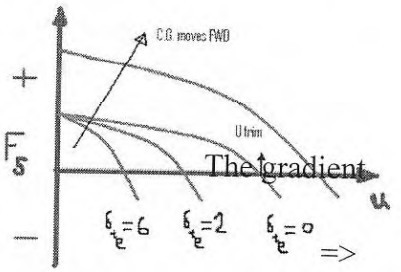
Substitute for $C_{h_t} = C_{h_{trim}}$ into equation (*) to obtain variation of Fs with U , other than U_{trim}

$$F_s = \eta_H G S_e C_e \left\{ \frac{-\bar{q}}{q_{trim}} \frac{W}{S} \frac{C_{h_\alpha}}{C_{m_\alpha}} * SM_{free} + \frac{w}{s} \frac{C_{h_\alpha}}{C_{m_\alpha}} SM_{free} \right\}$$

$$\frac{\partial F_s}{\partial V_p} = 0$$

$$U = U_{trim}$$

$$\left.\frac{\partial F_s}{\partial V_p}\right|_{U_{trim}} = -\frac{2}{U_{trim}} \eta_H S_e C_e \frac{W}{S} \frac{C_{h_\alpha}}{C_{m_\alpha}} SM|_{free} G$$



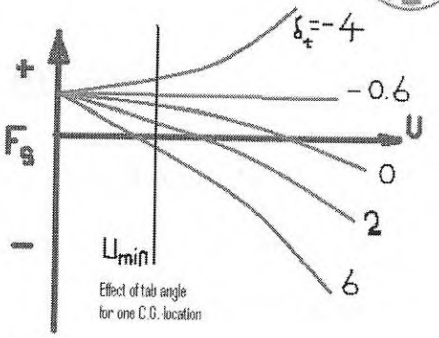
The gradient

Stick force v.s. speed gradient

(109_II)

The Gradient

- Decreases with trim speed => the trim ability deteriorates at higher speeds
- Proportional to W/S
- Depends on $\overline{X}_{cg}$
- Proportional to $\propto S_e.C_e$
- Varies with altitude



from bcfore $\left(\overline{X_{cg}} - \overline{X_{ac}}\right) = -\frac{C_{m_q}}{4W} \rho S C g$

Trim tab serves two purposes:

- Ability to zero-out the stick force
- Speed stability at the trim speed

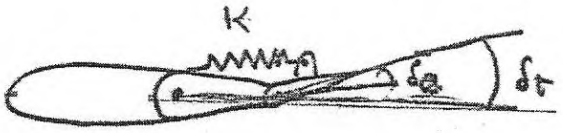
(110_I)

Its linkages must be such to permit tab deflection independent of elevator def. Use jack screw

Other kinds of tabs:

- Balance tab , mechanism to vary C_{h_δ} so to "stiffened" or "softened" the stick force gradient
- Blow-down tabs

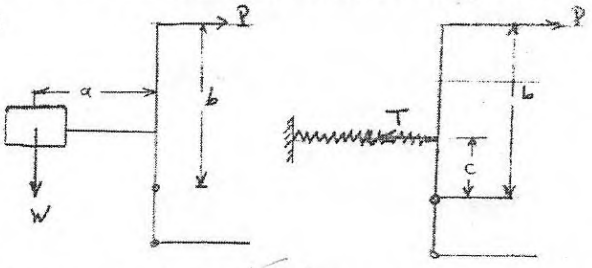
A way to increase stick-free static margin artificially by preloading the tab using a spring



(110_II)

Bob weights & springs

To modify the stick-force charact. of manual-control system use these systems



Both systems are designed so that it exerts a nearly const. force on the control system.

* both weight & spring req. an additive stick force Δp to maintain equilib.

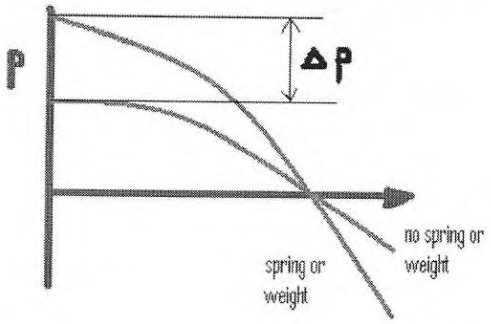
$\Delta p = w\ a/b$ for weight

$\Delta p = T\ c/d$ for spring

(111_I)

This addition is to improve longit. stability stick-free (most A/C have low gradients at the aft center of gravity)

To produce a steeper stick-force gradient at a given trim speed



(111_II)

Flight test meas. To obtain stick-free N.P.

$$\frac{F_s}{q} = GS_e C_e \eta_H \left(A + C_{h_\alpha}\delta_t\right) - S_e C_e \eta_H \frac{C_{h_\delta}}{C_{m_\delta}}\left(\frac{dC_m}{dC_L}\right)_{free} C_L$$

$$A = C_{h_0} + C_{h_\alpha}(\alpha_0 - i_w + i_t) + C_{h_\delta}\delta_{e_0}$$

$F_s = G * H.M.$ set $K = GS_e C_e \eta_H$

Diff. w.r.t. C_L $\quad \dfrac{d(F_s/q)}{dC_L} = -K\dfrac{C_{h_\alpha}}{C_{m_\delta}}\left(\dfrac{dC_m}{dC_L}\right)_{free}$

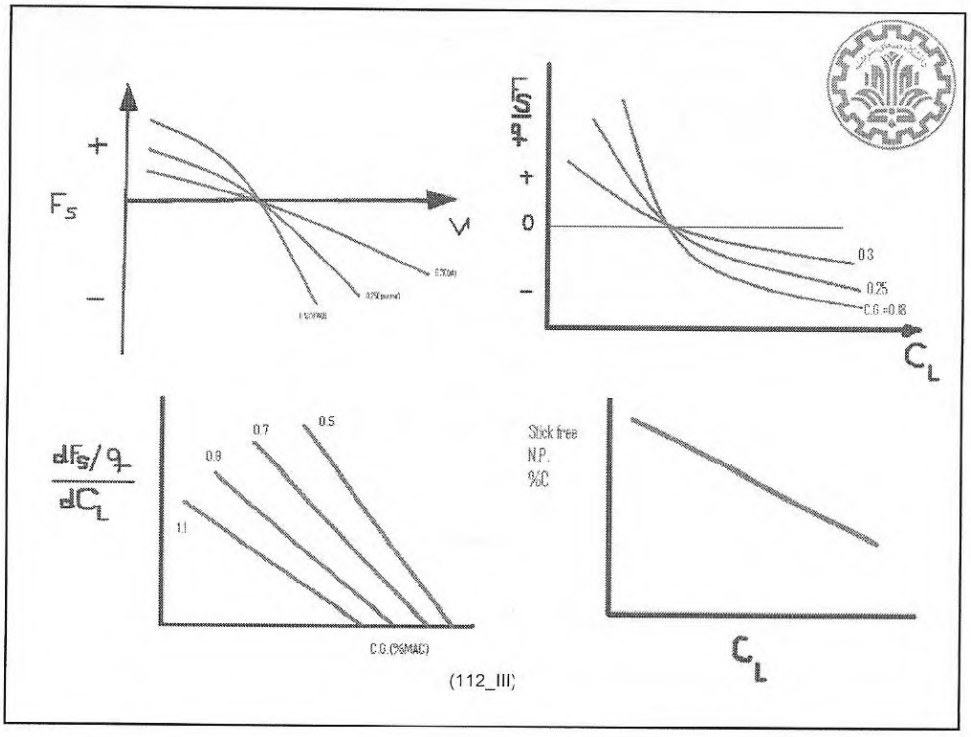
- variation of the slope of $\dfrac{F_s}{q}$ vs C_L is a direct fund. of $\left(\dfrac{dC_m}{dC_L}\right)_{free}$

(112_I)

Plot F_s vs V for various C.G.-

- Replot $\frac{F_S}{q}$ vs C_L for various C.G.
- Plot $\frac{d(F_s/q)}{dC_L}$ vs C.G.
- Interpolate or extrapolate C.G. for zero $\frac{d(F_s/q)}{dC_L}$

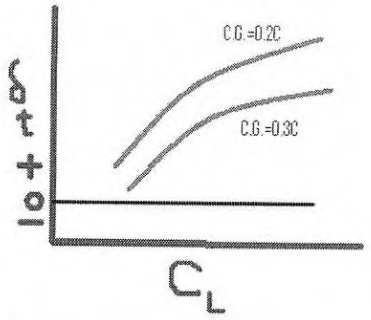
(112_II)



(112_III)

Second method

Obtain flight curves of tab angle to trim($F_s = 0$) vs V•
for varying C.G. location•
Slope δ_t vs C_L is a function of $\left(\frac{dC_m}{dC_L}\right)_{free}$ and the C.G. for a zero•
•
slope of $\frac{d\delta_e}{dC_L}$ is
also the stick-free
N.P.



(1·

Dynamic stability

Small Perturbation Equation

Assume that the motion of the A/C consists of small deviation about a steady flight condition.

No good for large-amplitude motions (spinning or stalled flight)
gives sufficient accuracy for practical engineering purposes.

* **Assume Variables = S.S. Value + Perturbed Value**

(114_I)

$U = U_1 + u \rightarrow u_1$ - S.S. value , u - perturbed value

$V = V_1 + v \qquad \Theta = \theta_1 + \theta$

$W = W_1 + w \qquad \Psi = \psi_1 + \psi$

•

•

$F_{A_X} = F_{A_{X_1}} + f_{A_X}$, etc.

Substitute into the equation of motions (3 force, 3 moments, 3 kinematics equation)

(114_II)

$$m(\dot{U} - VR + WQ) = -mg\sin\theta + F_{A_X} + F_{T_X}$$

$$m[(\dot{U}_1 + \dot{u}) - (V_1 + v)(R_1 + r) + (W_1 + w)(Q_1 + q)] =$$
$$-mg\sin(\theta_1 + \theta) + F_{A_{X1}} + f_{A_X} + F_{T_{X1}} + f_{T_X}$$

Assume θ and φ small

$\Rightarrow \cos\theta = \cos\varphi \approx 1, \quad \sin\theta \cong \theta, \quad \sin\varphi \cong \varphi$

$\Rightarrow \sin(\theta_1 + \theta) = \sin\theta_1\cos\theta + \cos\theta_1\sin\theta = \sin\theta_1 + \theta\cos\theta_1$

$$\Rightarrow m[\dot{u} - V_1R_1 - V_1r - vR_1 - vr + W_1Q_1 + W_1q + wQ_1 + wq]$$
$$= -mg(\sin\theta_1 + \theta\cos\theta_1) + F_{A_{X1}} + f_{A_X} + F_{T_{X1}} + f_{T_X}$$

•2nd order in perturbations are small: $vr \approx 0, \; wq \approx 0$

Steady State Equation.

(114_III)

$\therefore$ After canceling, we get

$$\Rightarrow m(\dot{u} - V_1 r - vR_1 + W_1 q + Q_1 w) = -mg\theta\cos\theta_1 + f_{A_X} + f_{T_X}$$

It is linear provided that f_{A_X}, and f_{T_X}

can be written as linear Equation. Book 2.78, 2.79, 2.82

Simplification: The reference flight condition is assumed to be symmetric with constant propulsive forces, No angular velocity.

$\Rightarrow$

1) $V_1 = 0$ (No Initial S.S side velocity)
2) $\phi_1 = 0$ (No Initial S.S bank angle)
3) $\omega = 0 \Rightarrow P_1 = Q_1 = R_1 = 0$ & $\dot{\psi}_1 = \dot{\theta}_1 = \dot{\varphi}_1 = 0$
(No Initial angular velocity)

(115-II)

Also align the x-axis so that it is along the direction of the A/C velocity vector.

- Substitute into the x-Equation:

$$\Rightarrow m(\dot{u} + W_1 q) = -mg\theta\cos\theta_1 + f_{A_X} + f_{T_X}$$
$$m(\dot{v} + U_1 r - W_1 p) = -mg\varphi\cos\theta_1 + f_{A_y} + f_{T_y} \quad (2.84)$$
$$m(\dot{w} - U_1 q) = -mg\theta\sin\theta_1 + f_{A_Z} + f_{T_Z}$$

$$\Rightarrow I_{xx}\dot{p} - I_{xz}\dot{r} = l_A + l_T$$
$$I_{yy}\dot{q} = m_A + m_T \quad (2.85)$$
$$I_{zz}\dot{r} - I_{xz}\dot{p} = n_A + n_T$$

(116-I)

$$\Rightarrow p = \dot{\varphi} - \dot{\psi}\sin\theta_1$$

$$q = \dot{\theta} \qquad (2.86)$$

$$r = \dot{\psi}\cos\theta_1$$

Above Equations are used for studying A/C dynamic stability, response to control & automatic flight control system.

Now consider the effect of perturbed motion variables u, v, w, p, q, r and their time derivatives on the perturbed aerodynamic forces and moments. (See 4.2)

(116-II)

$$F_{A_X} = \frac{\partial F_{A_X}}{\partial u}u + \frac{\partial F_{A_X}}{\partial w}w + \frac{\partial F_{A_X}}{\partial q}q + \frac{\partial F_{A_X}}{\partial \dot{w}}\dot{w}$$

$$+ \frac{\partial F_{A_X}}{\partial \alpha}\alpha + \frac{\partial F_{A_X}}{\partial \dot{\alpha}}\dot{\alpha} + \frac{\partial F_{A_X}}{\partial \delta_e}\delta_e + \frac{\partial F_{A_X}}{\partial \delta_T}\delta_T$$

q- Pitching velocity perturbation: $\alpha = \frac{w}{U_1}, \quad \beta = \frac{v}{U_1}$

Use quasi-steady assumption; perturbed forces and moments depend only on the instantaneous values of the motion and control variables and not on the time history of these variables.

(117-I)

- Write all differentiations w.r.t dimensionless variables

•Divide u by U_1

•Multiply •Longitudinal perturbed angular rates by $\frac{\bar{c}}{2U_1}$

•Lateral directional angular rates by $\frac{b}{2U_1}$

$$\frac{\partial F_{A_X}}{\partial u}u = \frac{\partial F_{A_X}}{\partial \frac{u}{U_1}}\frac{u}{U_1}; \qquad \frac{\partial F_{A_X}}{\partial \dot{\alpha}}\dot{\alpha} = \frac{\partial F_{A_X}}{\partial\left(\frac{\dot{\alpha}\bar{c}}{2U_1}\right)}\frac{\dot{\alpha}\bar{c}}{2U_1}$$

$$\frac{\partial F_{A_X}}{\partial q}q = \frac{\partial F_{A_X}}{\partial\left(\frac{q\bar{c}}{2U_1}\right)}\frac{q\bar{c}}{2U_1};$$

Table 4.7 lists all off these derivatives

(117-II)

Longitudinal Stability Derivatives:

Forces and moments affected by u are F_{A_X}, F_{A_y} and M_A

- Speed effects:

$$F_{A_X} = C_x\bar{q}S$$
$$F_{A_Z} = C_Z\bar{q}S$$
$$M_A = C_m\bar{q}S\bar{c}$$

All defined in the Stability axis

(117-III)

P.D of Equation w.r.t $\dfrac{u}{U_1}$

$$\left.\frac{\partial F_{A_X}}{\partial\left(\frac{u}{U_1}\right)}\right|_1 = \left.\frac{\partial C_X}{\partial\left(\frac{u}{U_1}\right)}\right|_1 q_1 S + C_{X1} S \left.\frac{\partial \overline{q}}{\partial\left(\frac{u}{U_1}\right)}\right|_1$$

Evaluated relative to a S.S

$$\left.\frac{\partial \overline{q}}{\partial\left(\frac{u}{U_1}\right)}\right|_1 = U_1 \frac{\partial}{\partial u}\left\{\left(U_1+u\right)^2+w^2+v^2\right\}\frac{1}{2}\rho\Big|_1 \qquad ;V_1 = W_1 = 0$$

$$\frac{\partial \overline{q}}{\partial\left(\frac{u}{U_1}\right)} = U_1\left\{\frac{1}{2}\rho\cdot 2\left(U_1+u\right)\right\}_1 = \rho U_1^{\,2}$$

(118-I)

When evaluated relative to a S.S

$C_X = -C_D\cos\alpha + C_L\sin\alpha$ Using small angles

$C_X = -C_D + C_L\alpha$

$$\frac{\partial C_X}{\partial\left(\frac{u}{U_1}\right)} = -\frac{\partial C_D}{\partial\left(\frac{u}{U_1}\right)} + \frac{\partial C_L}{\partial\left(\frac{u}{U_1}\right)}\alpha$$

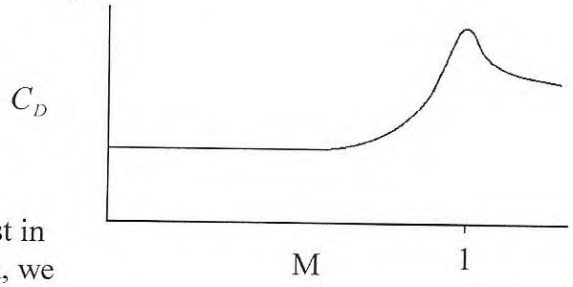
But $\alpha = 0$ at S.S reference condition

$$\Rightarrow \frac{\partial C_D}{\partial\left(\frac{u}{U_1}\right)} \equiv C_{D_U} \qquad \Rightarrow \frac{\partial C_X}{\partial\left(\frac{u}{U_1}\right)} \equiv -C_{D_U}$$

Can estimate C_{D_U} from a plot of the drag coefficient vs. M at constant α

(118-III)

$$C_{D_U} = \frac{\partial C_D}{\partial\left(\frac{u}{U_1}\right)} = \frac{U_1}{a}\frac{\partial C_D}{\partial\left(\frac{u}{a}\right)} = M_1\frac{\partial C_D}{\partial M}$$



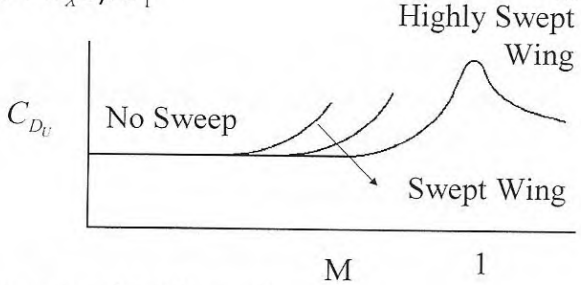
If include the thrust in the above equation, we get $\dfrac{\partial C_X}{\partial\left(\frac{u}{U_1}\right)} = -C_{D_U} + C_{T_U}$

$C_{T_U} = 0$ For gliding flight and a good appr. for jet A/C

(118-V)

For a variable pitch engine power plant, $\left(C_{T_U} \approx -C_D\right)$

$$\therefore \frac{\partial F_{A_X}}{\partial\left(\frac{u}{U_1}\right)} = -C_{D_U}\bar{q}_1 S + C_X S\rho U_1^{\,2}$$



At S.S condition when $\alpha = 0 \Rightarrow C_X = -C_D$

$$\Rightarrow \frac{\partial F_{A_X}}{\partial\left(\frac{u}{U_1}\right)} = -\left(C_{D_U} + 2C_{D_1}\right)\bar{q}_1 S$$

C_{D_U} - Speed damping derivative

(119-I)